Sets

Exercise - 1.1

1.
(i) Yes
(ii) NO
(iii) NO
(iv) Yes
(v) Yes
(vi) Yes
(vii) Yes
(viii) Yes
(ix) No

2.
(i) $5 \in A$
(ii) $8 \notin A$
(iii) $0 \notin A$
(iv) $4 \in A$
(v) $2 \in A$
(vi) $10 \notin A$

3.
(i) $A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{17, 26, 35, 44, 62, 71, 80\}$

(iv) $D = \{2, 3,\}$

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(v) $E = \{T, R, I, G, O, N, M, E, Y\}$

(vi) $F = \{B, E, T, R\}$

4.

(i) $A = \{x : x \quad 3n \quad n \in N \text{ and } 1 \le n \le 4\}$

(ii) $B = \{x : x \quad 2^n \quad n \in N \text{ and } 2 \le n \le 5\}$

(iii) $C = \{x : x \quad 5^n \quad n \in N \text{ and } 1 \le n \le 5\}$

(iv) $D = \{x : x$ is even natural no.$\}$

(v) $E = \{x : x \quad n^2 \quad n \in N \text{ and } 1 \le n \le 10\}$

5.

(i) $A = \{1, 3, 5, 7, 9, 11 \ldots\ldots\}$

(ii) $B = \{-0.4, -0.2, -0.1, 0, 1, 2, 3, 4\}$

(iii) $C = \{-2, -1, 0, 1, 2\}$

(iv) $D = \{L, O, Y, A\}$

(v) $E = \{$April, June, September, November, February$\}$

(vi) $F = \{b, c, d, f, g, h, j\}$

6.
(i) (c)
(ii) (a)
(iii) (d)
(iv) (b)

Exercise - 1.2

1.
~~(i) A = {~~

(i) { }
(iii) { }
(iv) { }

2. (i) Finite
(ii) Infinite
(iii) finite
(iv) Infinite
(v) Finite

3 (i) Infinite
(ii) Finite
(iii) Infinite
(iv) Finite
(v) Infinite

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4.

(i) $A = B$

(ii) $A \neq B$

(iii) $A = B$

(iv) $A \neq B$

5.

(i) $A = (2, 3)$ B

$$x^2 + 5x + 6 = 0$$
$$x^2 + 2x + 3x + 6 = 0$$
$$x(x+2) + 3(x+2) = 0$$
$$x = 3 = 0 \qquad x = 2 = 0$$
$$x = -3 \qquad x = -2$$

$A \neq B$

(ii) $A = \{F, O, L, W\}$
$B = \{W, O, L, F\}$

$A = B$

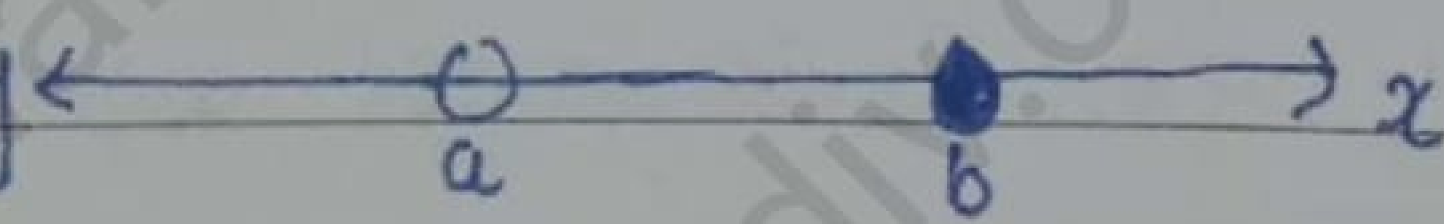
6.

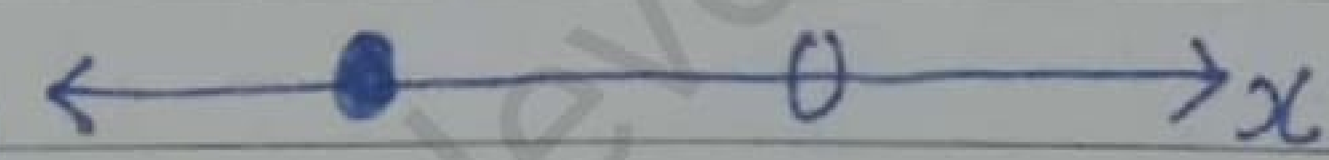
(i) $B = D$

(ii) $E = G$

$(a,b) \rightarrow \{x : a < x < b\}$


$[a,b] \rightarrow \{x : a \leq x \leq b\}$


$(a,b] \rightarrow \{x : a < x \leq b\}$


$[a,b) \rightarrow \{x : a \leq x < b\}$


Exercise -1.3

1. (i) $\subset$
(ii) $\not\subset$
(iii) $\subset$
(iv) $\not\subset$
(v) $\not\subset$
(vi) $\subset$
(vii) $\subset$

2. (i) False
(ii) True
(iii) False
(iv) True
(v) False
(vi) True

3.
(i) is incorrect because 3 and 4 are not present in A but the set $\{3,4\}$ is present.

(v) is incorrect because $1 \notin A$. Correct is $\{1\} \subset A$

(viii) is incorrect because $\{1,2,5\} \notin A$, Correct is $1,2,5 \in A$

(ix) is incorrect because $\phi \notin A$, Correct is $\phi \subset A$.

(xi) is incorrect because $\{\phi\} \notin A$, Correct is $\phi \subset A$

4.

(i) $\{a\}$, ϕ

(ii) $\{a\}$, $\{b\}$, $\{a,b\}$, ϕ

(iii) $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$, $\{1,2,3\}$, ϕ

(iv) ϕ

5. 1

6. (i) $[-4, 6]$
(ii) $(-12, -10)$
(iii) $[0, 7)$
(iv) $[3, 4]$

7.
(i) $\{x : x \in R, -3 < x < 0\}$
(ii) $\{x : x \in R, 6 \leq x \leq 12\}$
(iii) $\{x : x \in R, 6 < x \leq 12\}$
(iv) $\{x : x \in R, -23 \leq x \leq 5\}$

8. (i) The set of triangles
(ii) The set of triangles

9. (iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Exercise 1.4

1.

(i) $X \cup Y = \{1, 2, 3, 5\}$

(ii) $A \cup B = \{a, b, c, e, i, o, u\}$

(iii) $A \cup B = \{x : x = 1, 2, 4, 5$ or a multiple of $3\}$

(iv) $A \cup B = \{\cancel{x :} 2, 3, 4, 5, 6, 7, 8, 9\}$

(v) $A \cup B = \{1, 2, 3\}$

2. Yes, $A \subset B$
$A \cup B = \{a, b, c\}$

3. If $A \subset B$, then $A \cup B = B$

4.
(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
(iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$
(v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
(vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5.

(i) $X \cap Y = \{1, 3\}$

(ii) $A \cap B = \{a\}$

(iii) $A \cap B = \{3\}$

(iv) $A \cap B = \phi$

(v) $A \cap B = \phi$

6.

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = \phi$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \phi$

(vi) $A \cap (B \cup C) = \{7, 9, 11\}$

(vii) $A \cap D = \phi$

(viii) $A \cap (B \cup D) = \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\}$

(x) $(A \cup D) \cap (B \cup C) = \{7, 9, 11, 15\}$

7.

(i) $A \cap B = \{x : x$ is an even natural no.$\}$

(ii) $A \cap C = \{x : x$ is an odd natural no.$\}$

(iii) $A \cap D = \{x : x$ is a prime no.$\}$

(iv) $B \cap C = \phi$

(v) $B \cap D = \{2\}$

(vi) $C \cap D = \{x : x$ is an odd prime no.$\}$

8.

(iii) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$

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9.

(i) $A - B = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{3, 6, 9, 12, 18, 21\}$

(iv) $B - A = \{4, 8, 16, 20\}$

(v) $C - A = \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{5, 10, 20\}$

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$

(ix) $C - B = \{2, 6, 10, 14\}$

(x) $D - B = \{5, 10, 15\}$

(xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$

(xii) $D - C = \{5, 15, 20\}$

10.

(i) $X - Y = \{a, c\}$

(ii) $Y - X = \{f, g\}$

(iii) $X \cap Y = \{b, d\}$

11. The set of irrational no.

12.

(i) No, because $\{2, 3, 4, 5\} \cap \{3, 6\} \neq \phi$

(ii) No, because $\{a, e, i, o, u\} \cap \{a, b, c, d\} \neq \phi$

(iii) Yes, because $\{2, 6, 10, 14\} \not\subset \{3, 7, 11, 15\}$ and $\{2, 6, 10, 14\} \cap \{3, 7, 11, 15\} = \phi$

(iv) Yes, because $\{2, 6, 10\} \not\subset \{3, 7, 11\}$ and $\{2, 6, 10\} \cap \{3, 7, 11\} = \phi$

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Exercise - 1.5

1.

(i) $A' = \{5, 6, 7, 8, 9\}$
(ii) $B' = \{1, 3, 5, 7, 9\}$
(iii) $(A \cup C)' = \{7, 8, 9\}$
(iv) $(A \cup B)' = \{5, 7, 9\}$
(v) $(A')' = \{1, 2, 3, 4\}$
(vi) $(B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$

2.

(i) $A = \{a, b, c\} \Rightarrow A' = \{d, e, f, g, h\}$
(ii) $B = \{d, e, f, g\} \Rightarrow B' = \{a, b, c, h\}$
(iii) $C = \{a, c, e, g\} \Rightarrow C' = \{b, d, f, h\}$
(iv) $D = \{f, g, h, a\} \Rightarrow D' = \{b, c, d, e\}$

3.

(i) $\{x : x$ is an odd natural no.$\}$
(ii) $\{x : x$ is an even natural no.$\}$
(iii) $\{x : x \in N$ and x is not a multiple of 3$\}$
(iv) $\{x : x$ is a positive composite no. or $x = 1\}$
(v) $\{x : x \in N$ and x is not divisible by 3 or 5$\}$
(vi) $\{x : x \in N$ and x is not a perfect square$\}$
(vii) $\{x : x \in N$ and x is not a perfect cube$\}$
(viii) $\{x : x \in N$ and $x \neq 3\}$
(ix) $\{x : x \in N$ and $x \neq 2\}$
(x) $\{1, 2, 3, 4, 5, 6\}$
(xi) $\{x : x \in N$ and $x \subseteq 9/2\}$

4.

(i) $(A \cup B)' = A' \cap B'$

$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{1, 9\}$

$A' = \{1, 3, 5, 7, 9\}$

$B' = \{1, 4, 6, 8, 9\}$

$A' \cap B' = \{1, 9\}$

$\therefore (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

$A \cap B = \{2\}$

$(A \cap B)' = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A' = \{1, 3, 5, 7, 9\}$

$B' = \{1, 4, 6, 8, 9\}$

$A' \cup B' = \{1, 3, \cancel{4}, 5, 6, 7, 8, 9\}$

$\therefore (A \cap B)' = A' \cup B'$

6. A' will be the the set of equilateral triangles

7. (i) $A \cup A' = \underline{\ U\ }$

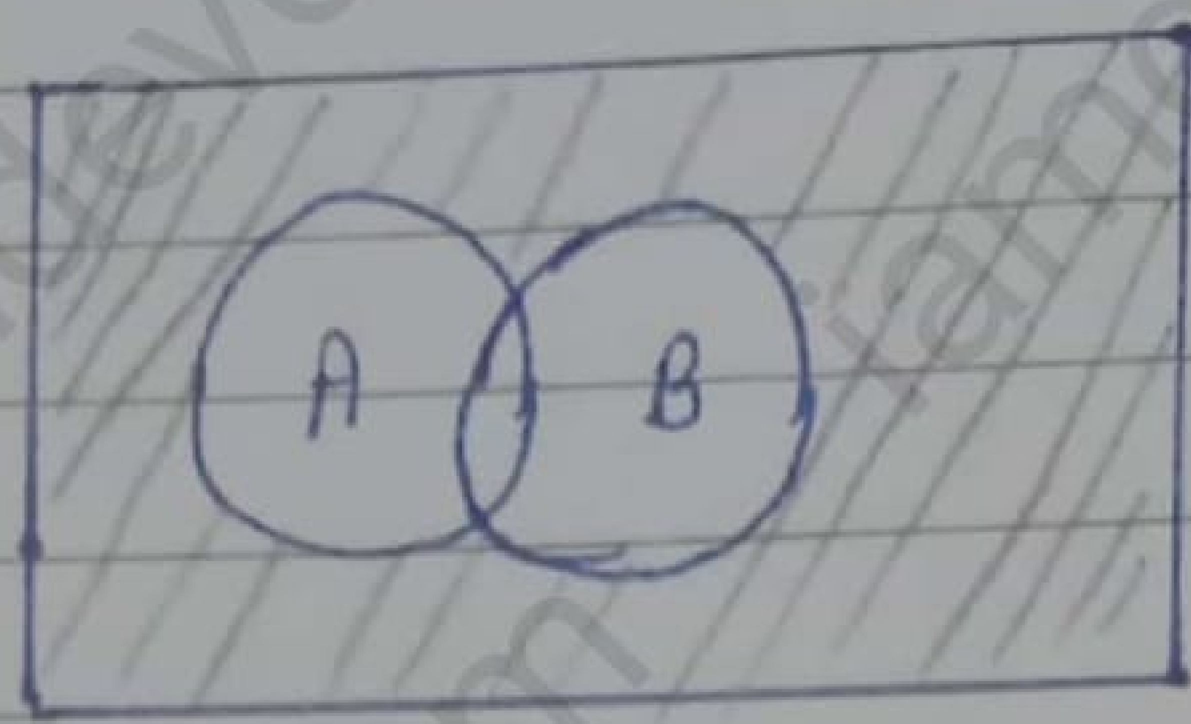
(ii) $\phi \cap A = \underline{\ A\ }$

(iii) $A \cap A' = \underline{\ \phi\ }$

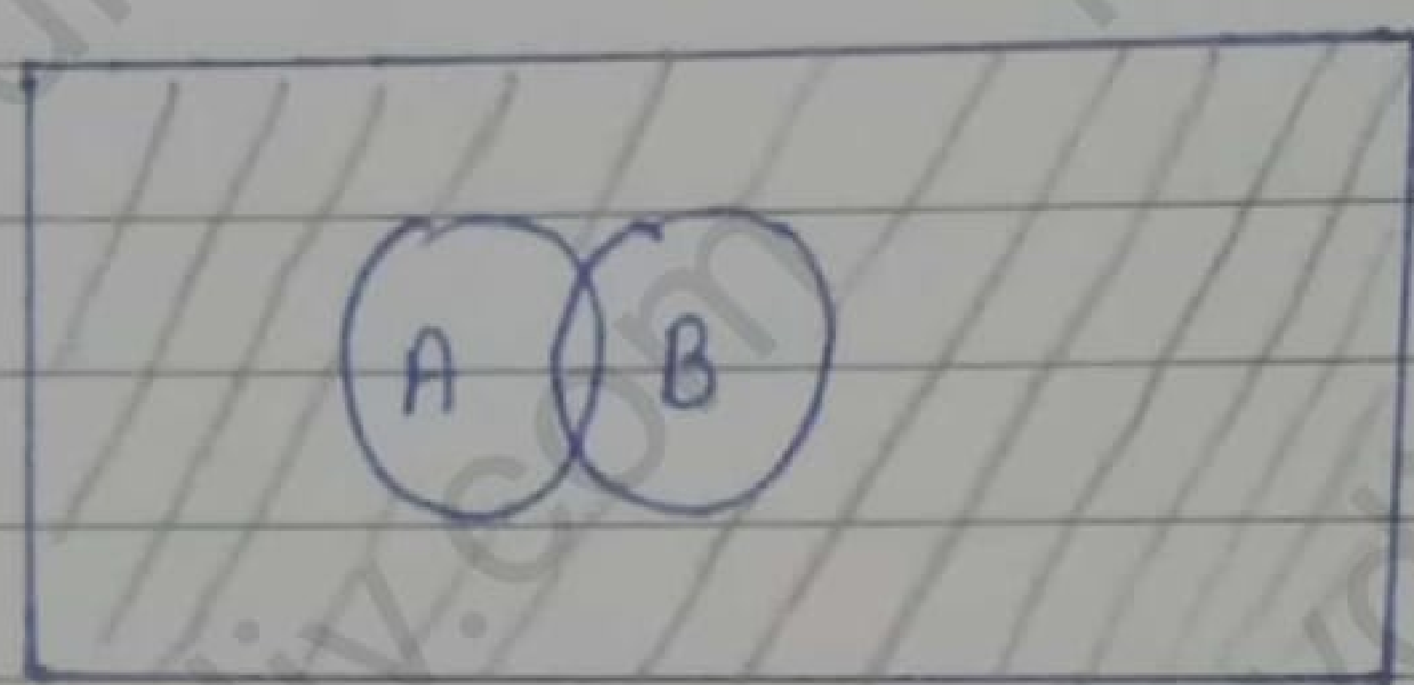
(iv) $U' \cancel{A} \cap A = \underline{\ \phi\ }$

5.

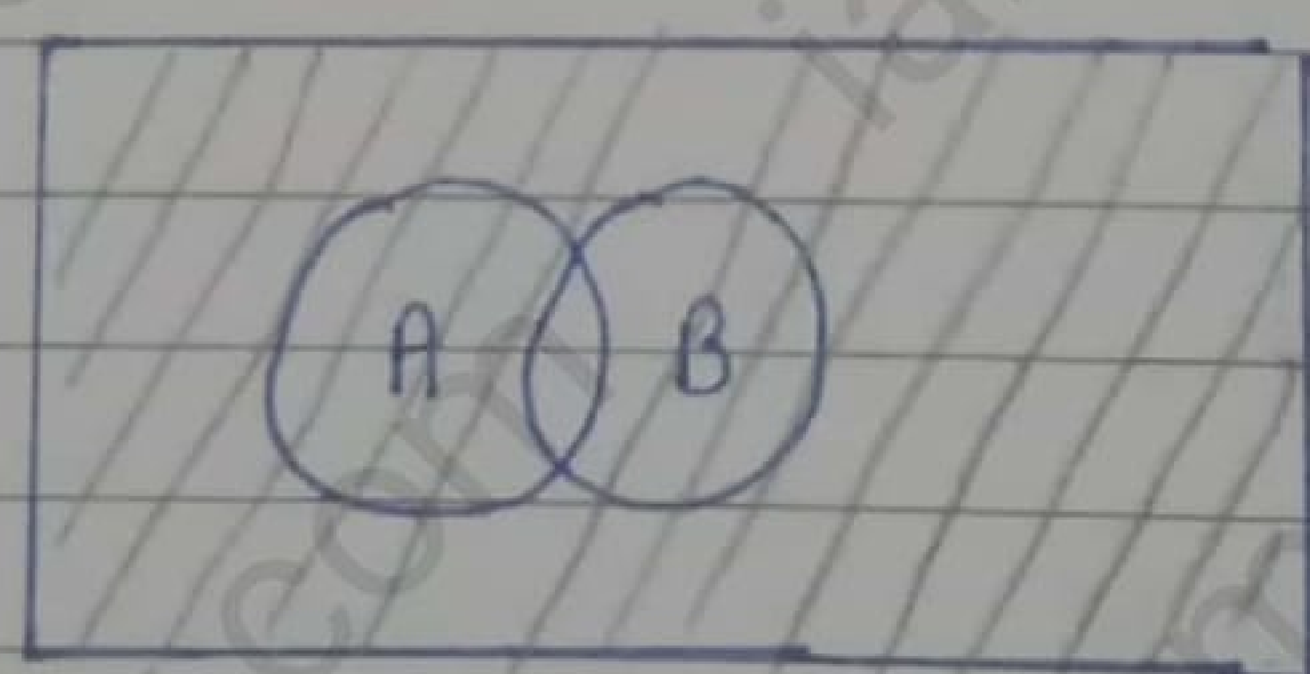
(i) $(A \cup B)'$



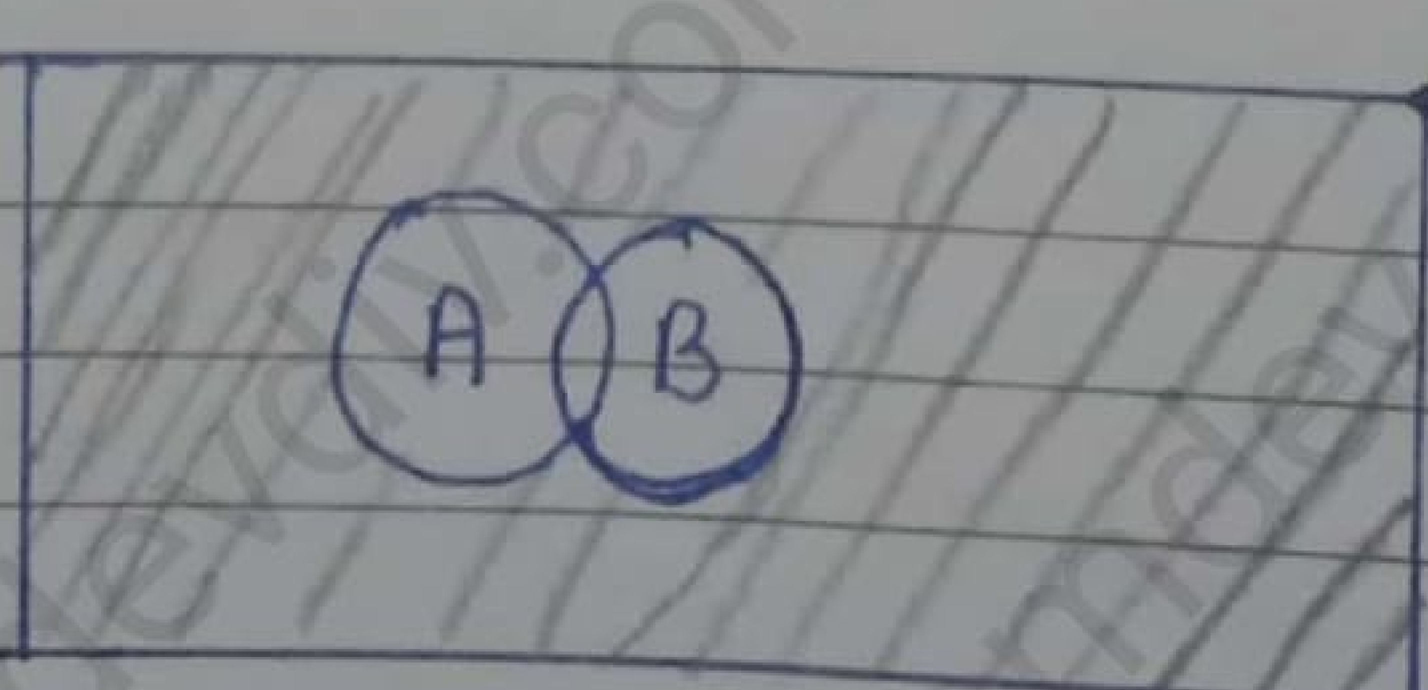
(ii) $A' \cap B' = (A \cup B)'$



(iii) $(A \cap B)'$



(iv) $A' \cup B' = (A \cap B)'$



Exercise - 1.6

$= n(A) + n(B) - n(A \cap B)$

~~Only A = n(A) = n(A ∪~~

1. $n(X) = 17$

$n(Y) = 23$

$n(X \cup Y) = 38$

$n(X \cap Y) = ?$

$n(X \cup Y) = \cancel{n(A) + n(B) - n(A \cap B)}$

$\cancel{38 = 1}$

Sol:- $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$38 = 17 + 23 - n(X \cap Y)$

$= 40 - n(X \cap Y)$

$n(X \cap Y) = 40 - 38$

$\boxed{n(X \cap Y) = 2}$

$n(X \cup Y) = 65$

$n(X) = 40$

$n(X \cap Y) = 10$

$n(Y) = ?$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$65 = 40 + n(Y) - 10$

$= 30 + n(Y)$

$65 - 30 = n(Y)$

$35 = n(Y)$

8. $n(X) = 50$

$n(Y) = 20$

$n(X \cap Y) = 10$

$n(X \cup X) = n(X) + n(X) - n(X \cap X)$

$= 50 + 20 - 10$

$= 70 - 10$

$n(X \cup X) = 60$

1.

A $\cancel{x^2 + 8x + 12 = 0}$

$\cancel{x^2 + 6x + 2x + 12 = 0}$

$\cancel{x(x+6)\ 2(x+6) = 0}$

$\cancel{x+6=0} \quad x+2=0$

$\cancel{x=-6} \quad x=-2$

$x^2 + 8x + 12 = 0$

$x^2 - 6x - 2x + 12 = 0$

$x(x-6) - 2(x-6) = 0$

$x - 2 = 0 \quad x - 6 = 0$

$x = 2 \quad x = 6$

$A = \{x : x \in R$ and x Satisfy $x^2 + 8x + 12 = 0\}$

$A = \{2, 6\}$

$B = \{2, 4, 6\}$

$C = \{2, 4, 6, 8 \cdots\}$

$D = \{6\}$

$A \subset B$, $A \in C$

$B \subset C$ $\quad D \subset C$

$D \subset A$

$D \subset B$

2.

(i) If $x \in A$ and $A \in B$, then $x \in B$

false

$x = 2 \in A$

$A = \{2\}$

$B = \{\{2\}, 3\}$

$\Rightarrow 2 \notin B$

$x \notin B$

(ii) If $A \subset B$ and $B \in C$, then $A \in C$

$A = \{2\}$

$B = \{2, 3\}$

$C = \{\{2, 3\}, 4\}$

$A \subset B$

$B \in C$

$A \notin C$ false

(iii) If $A \subset B$ and $B \subset C$, then $A \subset C$

Let $x \in A$

$A \subset B$

$\Rightarrow x \in B$

But $B \subset C$

$x \in C$

$\Rightarrow \boxed{A \subset C}$

(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$

$A = \{1, 2\}$
$B = \{1, 3, 5\}$
$C = \{1, 2, 3, 6, 7\}$

$A \subset C$ False

(v) If $x \in A$ and $A \not\subset B$ then $x \in B$

$x = 1$
$A = \{1, 2\}$
$B = \{3, 4, 5\}$

$x \notin B$ False

(vi) If $A \subset B$ and $x \notin B$ then $x \notin A$

$x = 4$
$A = \{1, 2\}$
$B = \{1, 2, 3\}$

$x \notin A$ True

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Some properties of Complement Sets

Prove that

$(A')' = A$

Let $x \in (A')'$

$\Rightarrow x \notin A'$

$\Rightarrow x \in A$

$\Rightarrow (A')' \subset A$ — (1)

Again Let $x \in A$

$x \notin A'$

$x \in (A')'$

$A \subset (A')'$ — (2)

from (1) & (2)

$(A')' = A$

<u>De Margan's Law</u>:-

(i) $(A \cup B)' = A' \cap B'$

Let $x \in (A \cup B)'$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A,\ x \notin B$

$x \in A',\ x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow (A \cup B)' \subset A' \cap B'$ — (1)

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Let $x \in A' \cap B'$
$\Rightarrow x \in A'$ & $x \in B'$
$\Rightarrow x \notin A$ & $x \notin B$
$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \in (A \cup B)'$
$\Rightarrow A' \cap B' \subset (A \cup B)'$ — (2)

From (1) & (2)
$\boxed{(A \cup B)' = A' \cap B'}$

(ii) $(A \cap B)' = A' \cup B'$

Let $x \in (A \cap B)'$
$\Rightarrow x \notin (A \cap B)$
$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \in A'$ or $x \in B'$
$\Rightarrow x \in (A' \cup B')$

$(A \cap B)' \subset A' \cup B'$ — (i)

Let $x \in A' \cup B'$
$x \in A$ or $x \in B'$
$x \notin A$ or $x \notin B$
$x \notin (A \cap B)$
$x \in (A \cap B)'$
$A' \cup B' \subset (A \cap B)$ — (ii)

from (i) & (ii)
$(A \cap B)' = A' \cup B'$

Complement Laws:-

(i) $A \cup A' = U$

$x \in A \cup A'$
$x \in A$ or $x \in A'$
$x \in A$ or $x \notin A$
$\boxed{A \cup A' = U}$

(ii) $\cancel{x \in}\ A \cap A' = \phi$

$x \in A \cap A'$
$\Rightarrow x \in A, x \in A'$
$\Rightarrow x \in A, x \notin A$
$\Rightarrow \boxed{A \cap A' = \phi}$

4.
(i) $\phi' = U$
Let $x \in \phi'$
$\Leftrightarrow x \notin \phi$
$\Leftrightarrow x \in U$
$\Rightarrow \boxed{\phi = U}$

(ii) $U' = \phi$

Let $x \in U'$
$\Leftrightarrow x \notin$

3.

Given:- $A \cup B = A \cup C$ - (1)

$A \cap B = A \cap C$ - (2)

To prove:- $B = C$

Proof:- from (1)

$(A \cup B) = (A \cup C)$

$(A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow (A \cap B) \cup (B \cap B) = (A \cap B) \cup (C \cap B)$

$\Rightarrow (A \cap B) \cup B = (A \cap B) \cup (B \cap C)$

$\Rightarrow B = (A \cap B) \cup (B \cap C)$ - 3

Again

$(A \cup B) = (A \cup C)$

$(A \cup B) \cap C = (A \cup C) \cap C$

$(A \cap C) \cup (B \cap C) = (A \cap C) \cup (C \cap C)$

$(A \cap B) \cup (B \cap C) = (A \cap C) \cup C$ [from 2]

$(A \cap B) \cup (B \cap C) = C$ - 4

from 3 & 4

$\boxed{B = C}$ Hence proved

Miscellaneous Exercise

Q 11. $A \cap X = B \cap X = \phi$

$\cancel{A \cap X = B}$ $A \cup X = B \cup X$

To prove:- $A = B$

$A = A \cap (A \cup X)$

$A = A \cap (B \cup X)$

$A = (A \cap B) \cup (A \cap X)$

$A = (A \cap B) \cup \phi$

$A = A \cap B$ - 1

$B = B \cap (B \cup X)$

$B = B \cap (A \cup X)$

$B = (B \cap A) \cup (B \cap X)$

$B = (B \cap A) \cup \phi$ — 2

from 1 & 2

$A = B$

8.

(i) $A = (A \cap B) \cup (A - B)$

Let $x \in (A \cap B) \cup (A - B)$

$\Rightarrow x \in (A \cap B)$ or $x \in (A - B)$

$\Rightarrow x \in A$ & $x \in B$ or $x \in A, x \notin B$

$\Rightarrow x \in A, x \in B$ or $x \notin B \Rightarrow x \in A$

$\Rightarrow (A \cap B) \cup (A - B) \subset A$ — 1

Again Let $x \in A$

$\Rightarrow x \in A, x \in B$ or $x \in B$

$\Rightarrow x \in A, x \in B$ or $x \in A, x \in B$

$\Rightarrow x \in A \cap B$ or $x \in A - B$

$\Rightarrow x \in (A \cap B) \cup (A - B)$

$\Rightarrow A \subset (A \cap B) \cup (A - B)$ — 2

From 1 & 2

$$\boxed{A = (A \cap B) \cup (A - B)}$$

Hence proved

(ii) $A \cup (B - A) = (A \cup B)$

$= A \cup (B \cap A')$
$= (A \cup B) \cap (A \cup A')$
$= (A \cup B) \cap U$
$= A \cup B = R.H.S.$

9.
(i) $A \cup (A \cap B) = A$

L.H.S.
$A \cup (A \cap B)$
$= (A \cup A) \cap (A \cup B)$
$= A \cap (A \cup B)$
$= A = R.H.S$

(ii) $A \cap (A \cup B) = A$

$A \cap (A \cup B)$
$(A \cap A) \cup (A \cap B)$
$A \cup (A \cup B)$
$A = R.H.S.$

10.

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5\}$

$C = \{3, 4, 7\}$

$A \cap B = \{3, 4\}$

$A \cap C = \{3, 4\}$

$B \neq C$

Hence $A \cap B = A \cap C$ need not imply $B = C$

12.

$A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5\}$
$C = \{4, 5, 6\}$

$A \cap B = \{3, 4\}$
$B \cap C = \{4, 5\}$

$A = \{1, 2\}$
$B = \{2, 3\}$
$C = \{3, 1\}$

$A \cap B = \{2\}$
$B \cap C = \{3\}$
$A \cap C = \{1\}$

$$\boxed{A \cap B \cap C = \phi}$$

Hence proved

3. Let - - - - - - - - - - - - C?

Given:- $A \cup B = A \cup C$ — ①
$A \cap B = A \cap C$ — ②

To prove :- $B = C$

Proof :- Three sets A, B, C

from eq. ①

$$(A \cup B) = (A \cup C)$$
$$(A \cup B) \cap B = (A \cup C) \cap B$$
$$(A \cap B) \cup (B \cap B) = (A \cap B) \cup (C \cap B)$$
$$(A \cap B) \cup B = (A \cap B) \cup (B \cap C)$$
$$B = (A \cap B) \cup (B \cap C) \quad - ③$$

Again

$$(A \cup B) = (A \cup C)$$
$$(A \cup B) \cap C = (A \cup C) \cap C$$
$$(A \cap C) \cup (B \cap C) = (A \cap C) \cup C$$
$$(A \cap B) \cup (B \cap C) = C \quad - ④$$

From eq ③ & ④

$B = C$ Hence proved