10/10/2023

# Gravitation

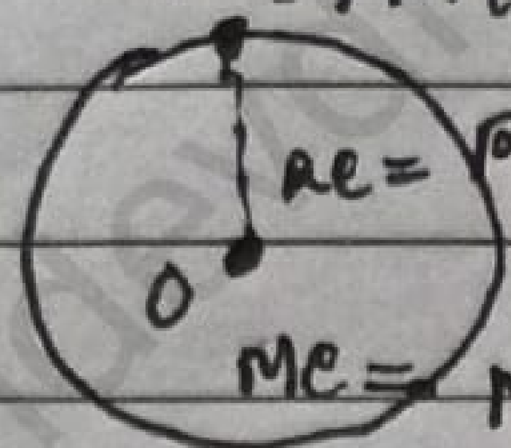
- # gravity & gravitational force
- # Newton gravitational force / Law
- # Kepler's Law (statement formula)
- # a value of g at h height from earth surface
- # b value of g at depth from " "
- # c value of g at equator & pole (Only formula)
- # (a + b + c) Numericals
- # 5 Escape speed complete
- # orbital speed
- # Gravitation potential Energy # Relation b/w ve & vo.
- # 4 Satellite (Only for case study)
- # NCERT Numericals, MCQ
- # Conceptual Questions

Re - Radius of Earth
Me - Mass of Earth

# $g = 9.8 \, m/s^2 \rightarrow 980 \, cm/s^2$
# $G = 6.67 \times 10^{-11} \, Nm^2/kg^2 \rightarrow 6.67 \times 10^{-8} \, dyne \, cm^2 g^2$

NGL

P → m(object)




$R_e$ = Radius of earth
= 6400 km = $64 \times 10^5$ m

$M_e$ = Mass of earth
= $6 \times 10^{24}$ kg

$$F \propto M_e m \quad -(1)$$

$$F \propto \frac{1}{R_e^2} \quad -(2)$$

from eq (1) & (2)

$$F \propto \frac{M_e m}{R_e^2}$$

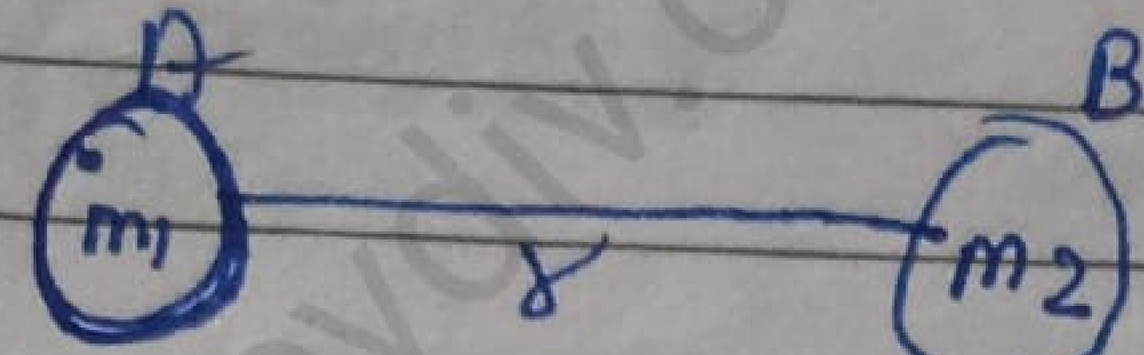
$$F = G \frac{M_e m}{R_e^2} \quad -(3)$$

G = Universal Gravitational Constant

$= 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$ mks

Note → $= 6.67 \times 10^{-11} \times 10^5$ dyne $\times 10^4\ cm^2 \times 10^{-6}\ gm^{-2}$
$= 6.67 \times 10^{-17+9}$
$= 6.67 \times 10^{-8}$ dyne $cm^2\ gm^{-2}$ (cgs)

# Note




$$F \propto m_1 m_2 \quad - (1)$$

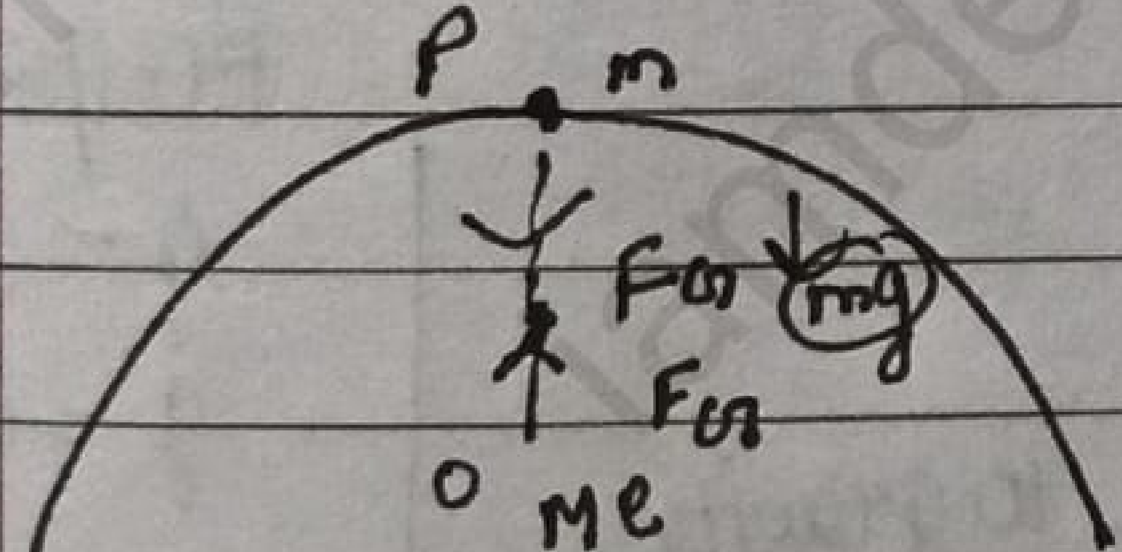
$$F \propto \frac{1}{r^2} \quad - (2)$$

from (1) & (2)

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

#




$$F_G = G\frac{M_e m}{R_e^2} \quad - (1) \qquad F_g = mg \quad - (2)$$

$$F_G = F_g$$

$$G\frac{M_e \cancel{m}}{R_e^2} = \cancel{m}g \rightarrow \boxed{g = G\frac{M_e}{R_e^2}}$$

U.G.G

$$g = \frac{G M_e}{R_e^2}$$

$M_e \rightarrow$ ~~6.67~~ Mass of earth ($6 \times 10^{24}$ kg)

$R_e^2 \rightarrow$ ~~$6 \times 10^{24}$ kg~~ 6400 km (Radius of Earth)

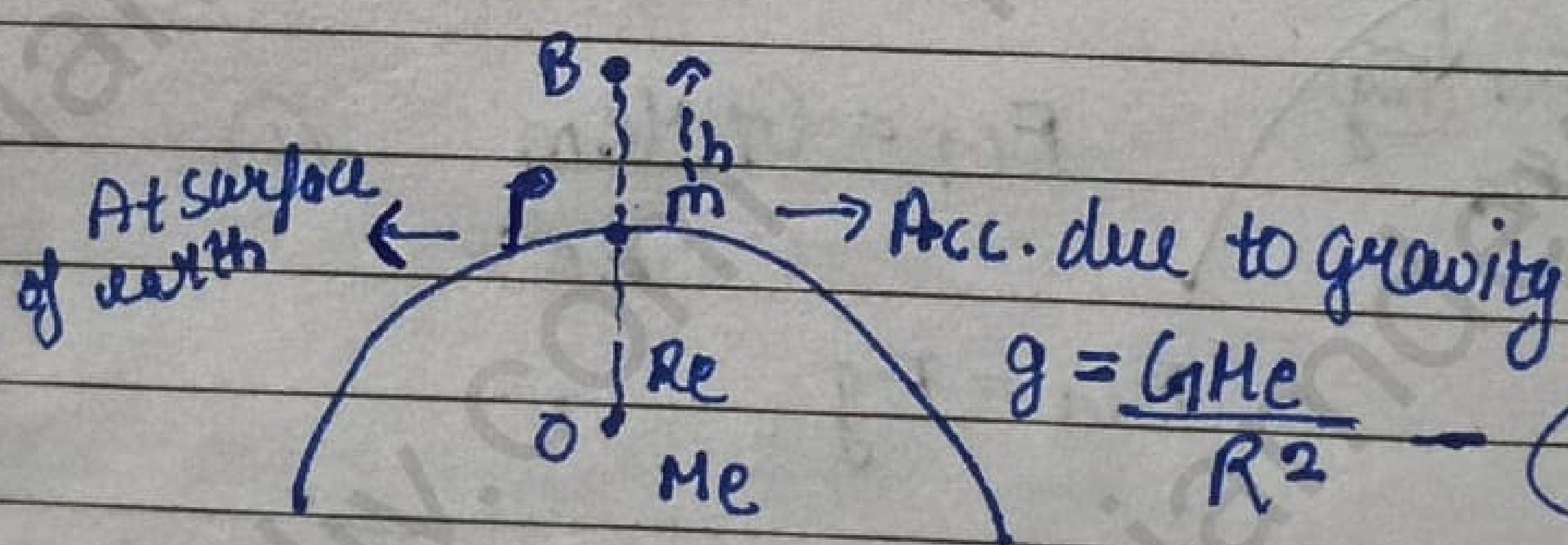
$g$: Acc. due to gravity ($9.8 \text{ m/s}^2$)

## Points

- Re - Radius of Earth
- Me - Mass of Earth
- G - $6.67 \times 10^{-11}\ Nm^2 kg^{-2}$ (mks)
  $6.67 \times 10^{-8}$ dyne $cm^2 gm^{-2}$ (cgs)
- $F = \frac{G M_e m}{R_e^2}$
- $g = G \frac{M_e}{R_e^2}$

# Variation of Acceleration due to gravity

Case 1. Effect of altitude / At h height from earth surface




$g = \frac{GM_e}{R^2}$ — (1)

Helping key
g - G
$g = \frac{GM_e}{R_e^2}$
On Earth surface
B.T.
$(1+x)^n = (1+nx)$
$(1+x)^{-n} = (1-nx)$

$\boxed{\begin{aligned} OB &= OP + BP \\ OB &= R_e + h \end{aligned}}$

$g \propto M_e$

$g \propto \frac{1}{R_e^2}$

$G = $ Constant

Because we know that Acc. due to gravity at Earth's surface point P $g = \frac{GM_e}{R^2}$ — (1)

Acc. due to gravity at point B , $g' = \frac{GMe}{BO^2}$

$$g' = \frac{GMe}{(Re+h)^2} \quad -(2)$$

eq (2) / (1)

$$\frac{g'}{g} = \frac{\frac{GMe}{(Re+h)^2}}{\frac{GMe}{Re^2}} = \frac{Re^2}{(Re+h)^2}$$

$$\frac{g'}{g} = \frac{Re^2}{Re^2\left(1+\frac{h}{Re}\right)^2} = \frac{1}{\left(1+\frac{h}{Re}\right)^2} = \left(1+\frac{h}{Re}\right)^{-2} = \underbrace{\left(1-\frac{2h}{Re}\right)}_{B.T.}$$

$$\frac{g'}{g} = \left(1-\frac{2h}{Re}\right)$$

$$g' = g\left(1-\frac{2h}{Re}\right) \quad -(3)$$

$$\left(1-\frac{2h}{Re}\right) < 1$$

So $g' < g$ ↓ value decrease

Ques At what height acc. due to gravity reduce 64%?

$R_e = 6400 km$

$g' = \frac{36}{100} g$

$g' = g \frac{R_e^2}{(R_e+h)^2}$

$= g \frac{(6400)^{\cancel{2}}}{(6400+h)^2}$

$\frac{36}{100} g = g \left(\frac{R_e}{R_e+h}\right)^2$

$\frac{36}{100} = \frac{9.8 \times 6400 \times 6400}{(6400+h)^2}$

$= \left(\frac{6}{10}\right)^{\cancel{2}} = \left(\frac{R_e}{R_e+h}\right)^{\cancel{2}}$

$\frac{36}{100} = 9.8 \times 6$

$= 6R_e + 6h = 10 R_e$

$\Rightarrow {}^{3}\cancel{6}h = {}^{2}\cancel{4}R_e$

$\Rightarrow 3h = 2R_e$

$= h = \frac{2 \times 6400}{3}$

$\Rightarrow \frac{12800}{3} = 4266.66$

$= 4267$ (Ans)

Ques- At what height from the earth surface the acc. due to gravity ~~stow~~ reduce 36% of its initial value of earth surface?

$R_e = 6400 km$

$g' = \frac{64}{100} g$

$$g' = g\frac{R_e^2}{(R_e+h)^2}$$

$$\frac{64}{100}g = g\left(\frac{R_e}{R_e+h}\right)^2$$

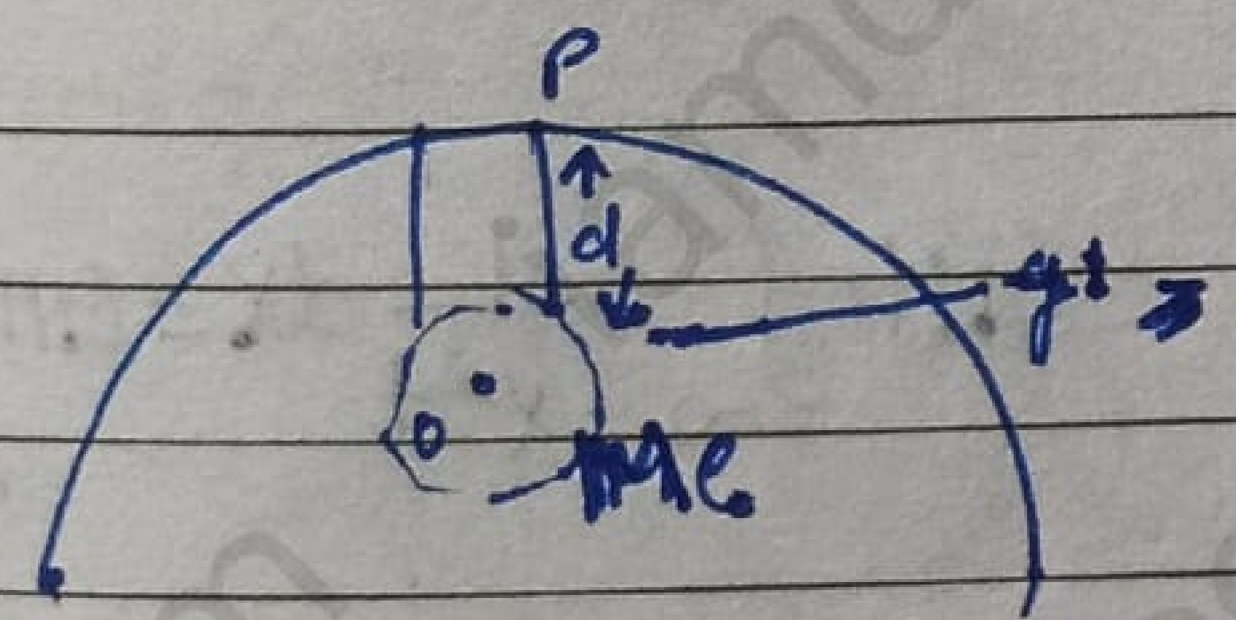
$$= \left(\frac{8}{10}\right)^2 = \left(\frac{R_e}{R_e+h}\right)^2$$

$$\cancel{= 8R_e + 8h = 10R_e}$$
$$\cancel{= 4h = R_e}$$
$$\cancel{= 2h = R_e}$$
$$\cancel{= h = \frac{6400}{4}}$$

$$= 8R_e + 8h = 10R_e$$
$$= 8h = 2R_e$$
$$= 4h = R_e$$
$$= h = \frac{6400}{4} = 1600$$

$$= h = 1600\,km \text{ (Ans)}$$

#




$M_e = Vol. \times density$
$= \frac{4}{3}\pi R_e^3 \rho$

$$g = \frac{GM_e}{R_e^2} = G\frac{\frac{4}{3}\pi R_e^3 \rho}{R_e^2}$$

$OP = R_e$
$PB = d$
$\boxed{OB = R_e - d}$

$$g = \frac{4}{3}\pi G \rho R_e \quad -(1)$$

$$g' = \frac{4}{3}\pi G \rho (R_e - h) \quad -(2)$$

(2) / (1)

$$\frac{g'}{g} = \frac{R_e - h}{R_e} = \left(1 - \frac{h}{R_e}\right)$$

$$\boxed{g' = g\left(1 - \frac{h}{R_e}\right)}$$

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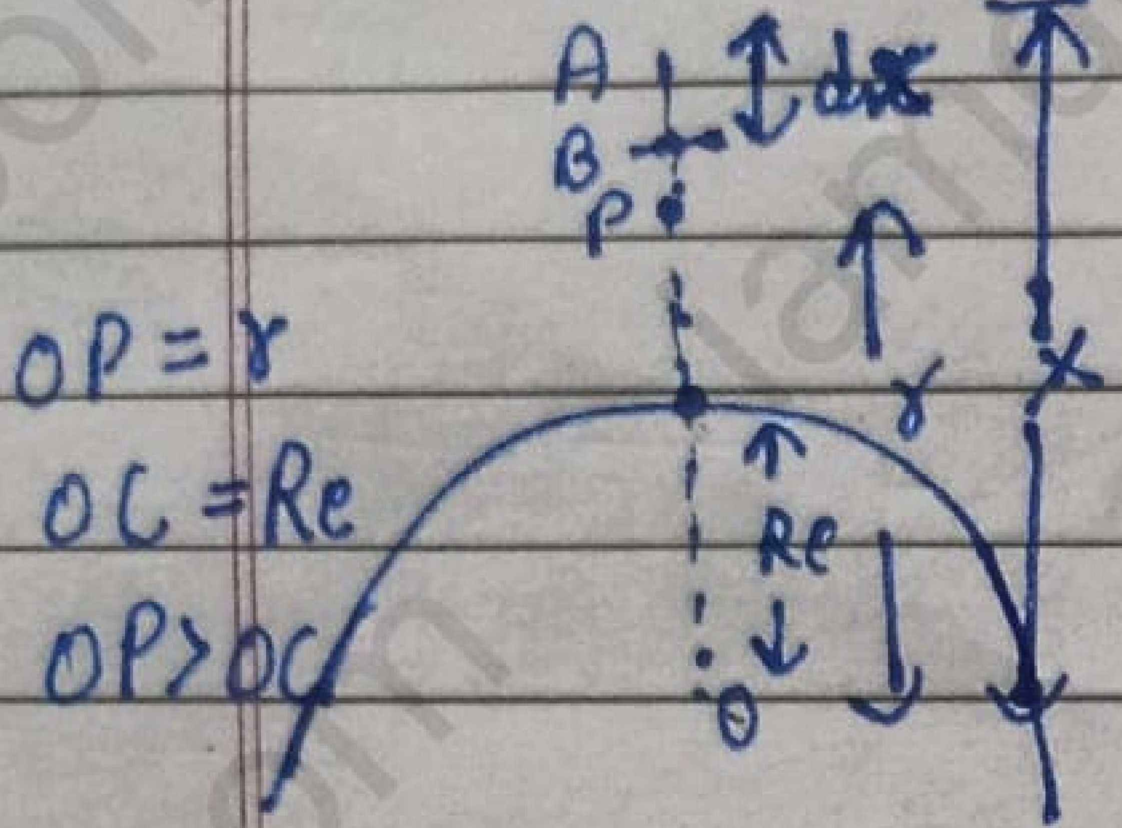
# Gravitational potential $V_P$ or $P$

↳ $J/kg \rightarrow [M^0L^2T^{-2}]$

$\infty, m_0 = 1$ $\infty$ to $r$ object → $m_0$ = unit mass




Gravitational force b/w A & O

$$F = G\frac{M_e m_0}{x^2} = \frac{GM_e \times 1}{x^2} = \frac{GM_e}{x^2} \quad (1)$$

To carry unit mass from A to B required work done = $dW = F dx$ — (2)

$$dW = G\frac{M_e}{x^2}dx \quad (3)$$

For $\infty$ to $r$, required work done.

$$\int_0^W dW = \int_\infty^r \frac{GM_e}{x^2}dx = GM_e\int_\infty^r \frac{1}{x^2}dx = GM_e\int_\infty^r x^{-2}dx$$

$$W = GM_e\left[-\frac{1}{x}\right]_\infty^r = -GM_e\left[\frac{1}{r} - \frac{1}{\infty}\right] = -\frac{GM_e}{r}$$

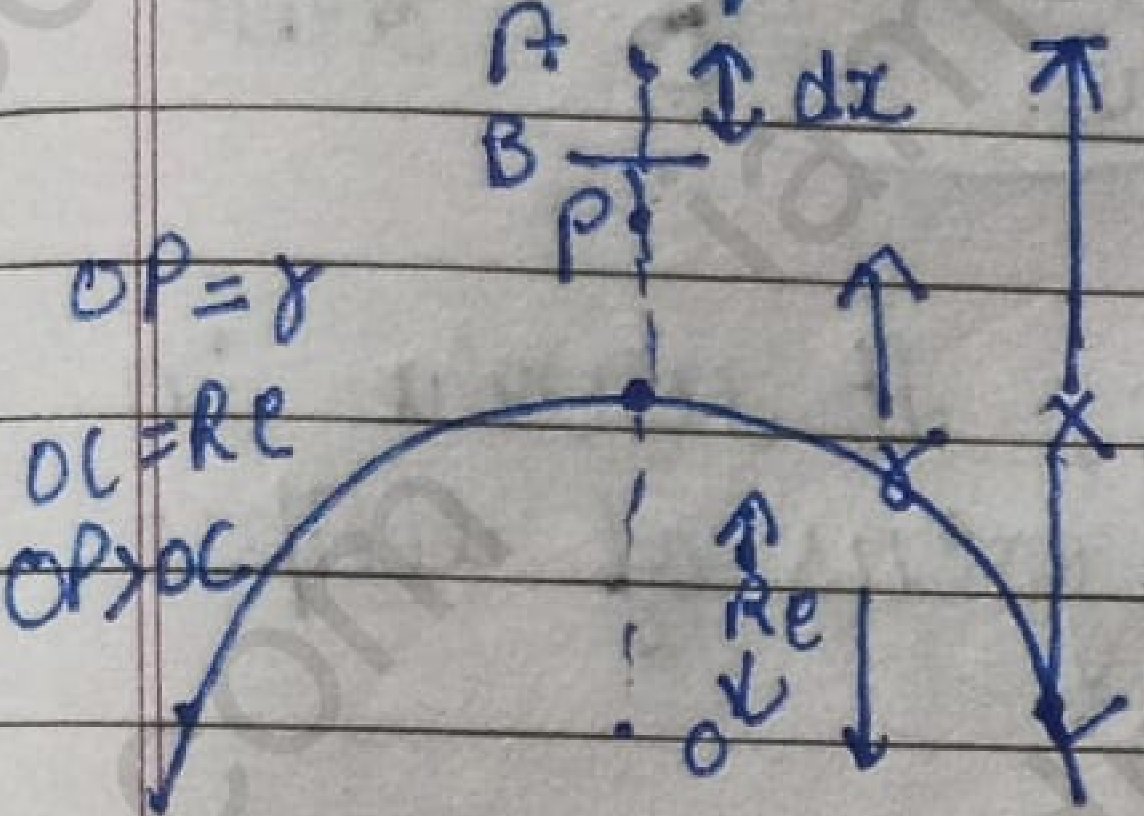
Work done per unit mass $\boxed{V_P = \frac{W}{m} = -\frac{GM_e}{r}}$

# Gravitational potential Energy P.E or G.P.E.

$\infty$ to $r$

mass object $\rightarrow m$

Required ~~...~~ work done / Stored Energy = G.P.E.

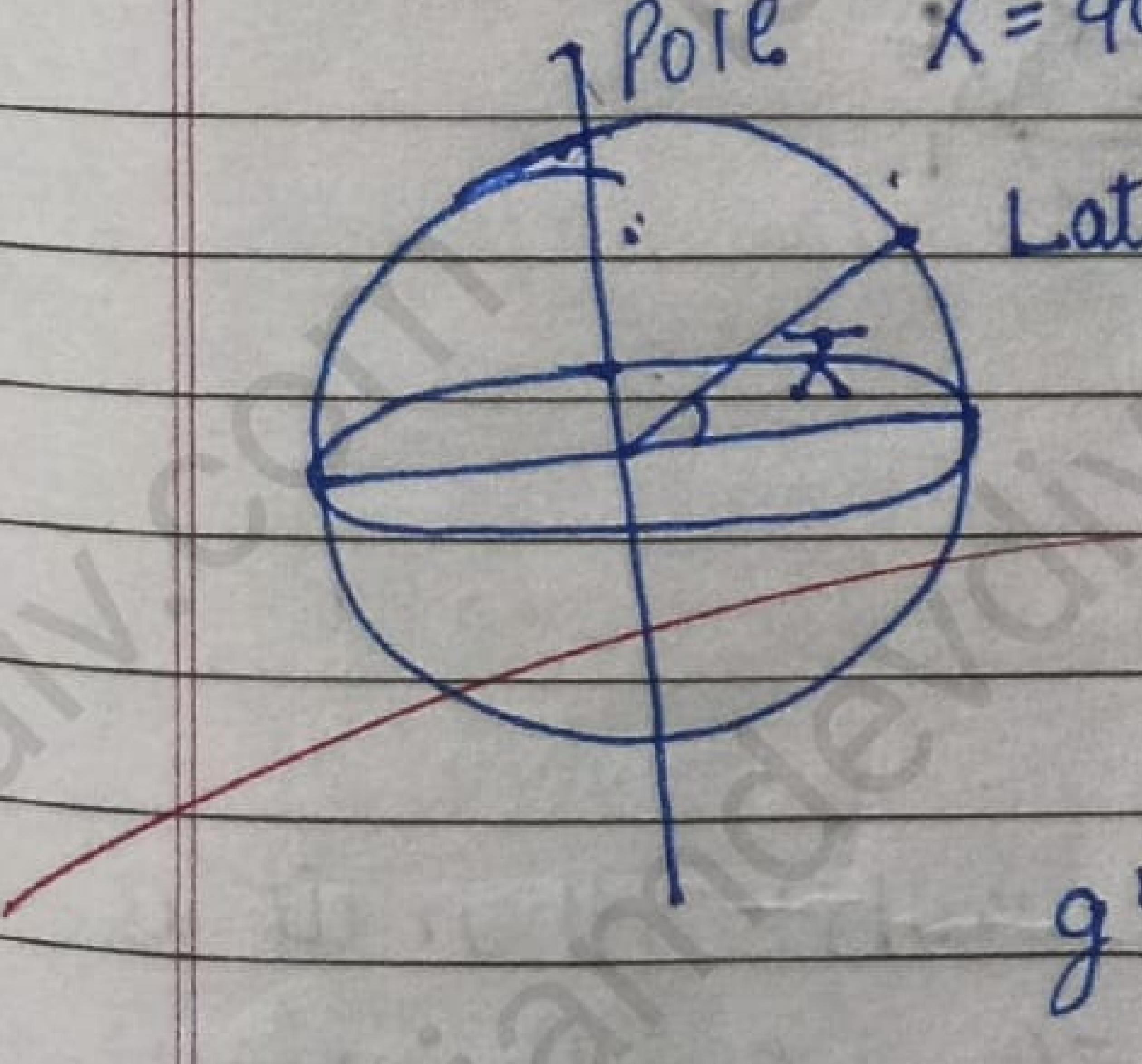



$$F = \frac{G M_e m}{x^2}$$

$$W = -\frac{G M_e m}{r}$$

G.P.E. = GP × mass of object

Write down the variation in value of 'g' at equator and pole of earth




equator
$x = 0$

At poles $\Rightarrow \lambda = 90^\circ$
$\cos 90^\circ = 0$

$$g' = g - R\omega^2 \cos^2 \lambda$$

$$g_P = \cancel{R} = g - R\omega^2 (\cos 90)^2 = g$$

$$g_e = g - R\omega^2$$

$g' = g - R\omega^2 \times 0$

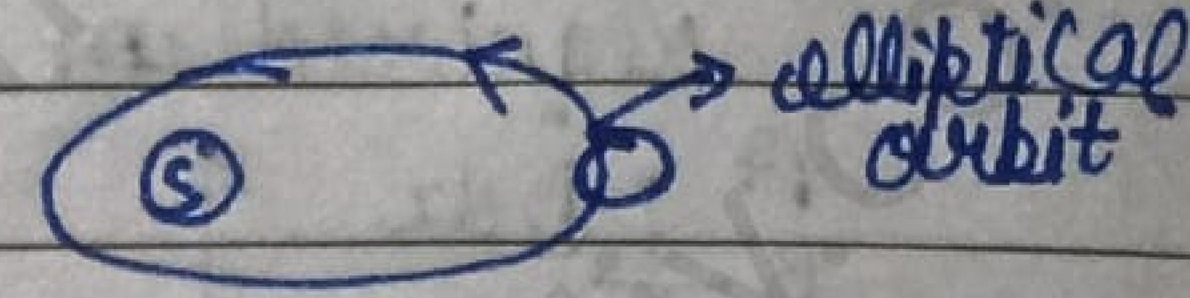
$\boxed{g' = g}$

At equator $\Rightarrow \lambda = 0^\circ \Rightarrow (\cos 0^\circ)^2 = (1)^2 = 1$

$$\boxed{\begin{aligned} g' &= g - R\omega^2 \times 1 \\ g' &= g - R\omega^2 \end{aligned}}$$

Write down the variation in value of g at equator and pole of earth

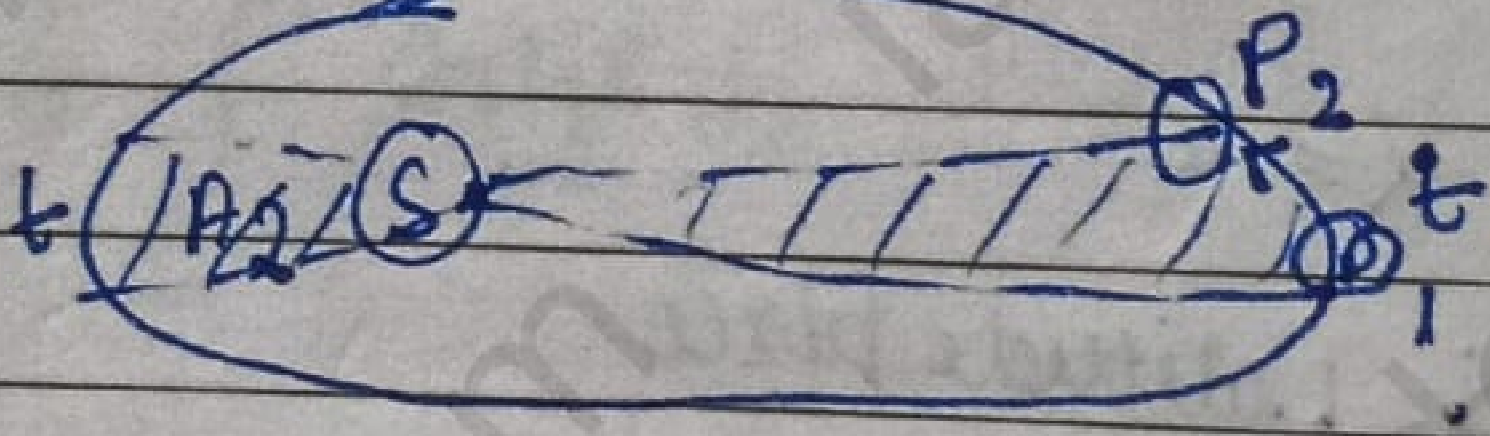
Kepler's Law.




1. $1^{st}$ Law
when planet revolve around sun, it revolve in elliptical orbit with sun at one focus of ellipse.

2. $II^{nd}$ Law

when a planet revolve in its orbit planet cover equal area in equal time interval i.e., arial velocity of planet is constant in its orbit i.e., Area covered per unit time by planet in its orbit is constant & this is known as arial velocity.




$A_1 = A_2$

Proof of 2nd law

when a planet revolve in its orbit then it is case of isolated system i.e. $\tau_{ex} = 0$ i.e., Angular momentum of planet is constant in its orbit

# By Geometric meaning of A. M., we know.

$$L = 2m.\frac{dA}{dt} = constant$$

$\because$ m is constant

$$\therefore \frac{dA}{dt} = \text{constant}$$

i.e. $\frac{\Delta A}{\Delta t} = \text{constant}$

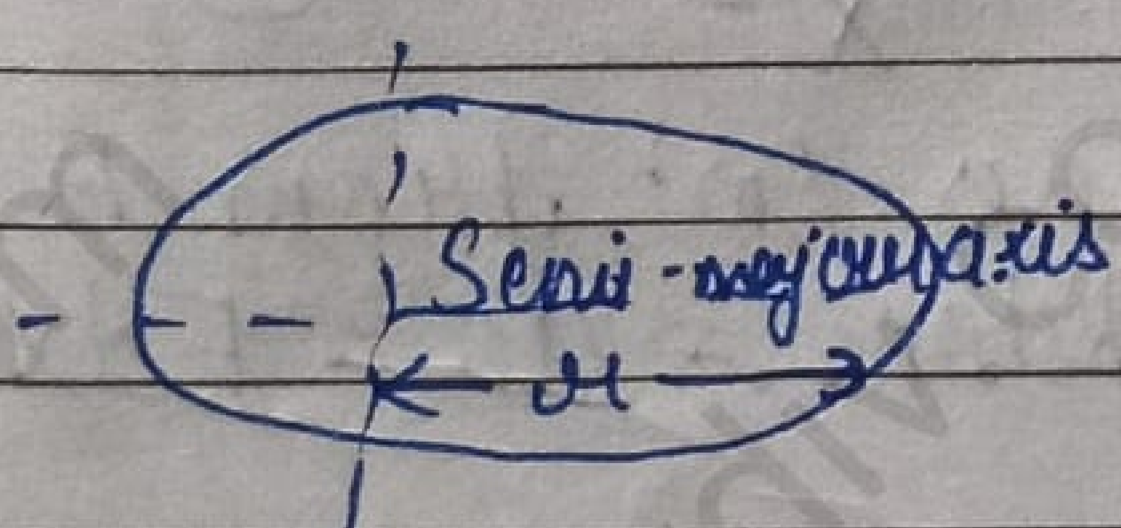
$\frac{A}{t} = \text{Constant} = \text{arial velocity}$

arialvelocity = constant

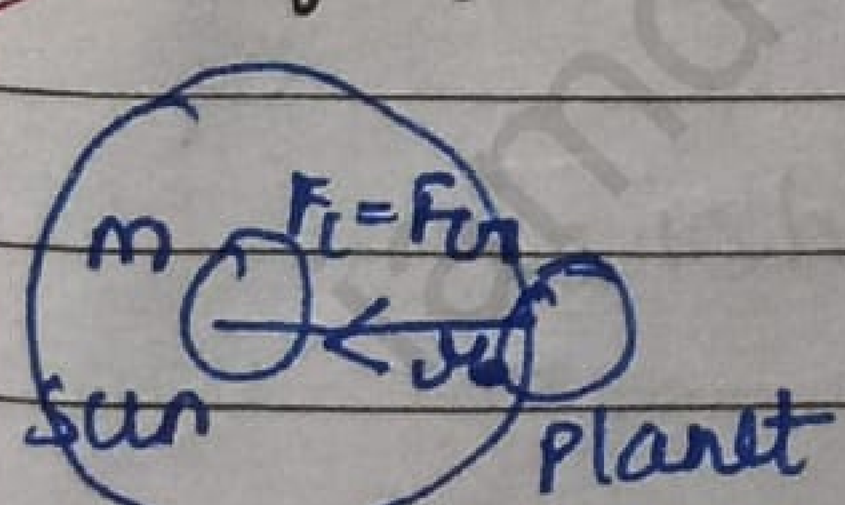
# Kepler's Third Law

$$T^2 \propto r^3$$

Acc. to Kepler's IIIrd law, square of time period of revolution of planet around sun is directly propotional to ~~the~~ cube of semi-major axis




# Proof of Kepler's 3rd Law




here, $F_G = F_C$

$$\frac{GmM}{r^2} = Mr\omega^2$$

$$\omega^2 = \frac{Gm}{r^3}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{Gm}{r^3}$$

$$\frac{4\pi^2}{T^2} = \frac{Gm}{r^3}$$

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

$$\frac{4\pi^2}{Gm} = \text{Constant} = K$$

$$T^2 = Kr^3$$

$$\boxed{\therefore T^2 \propto r^3}$$

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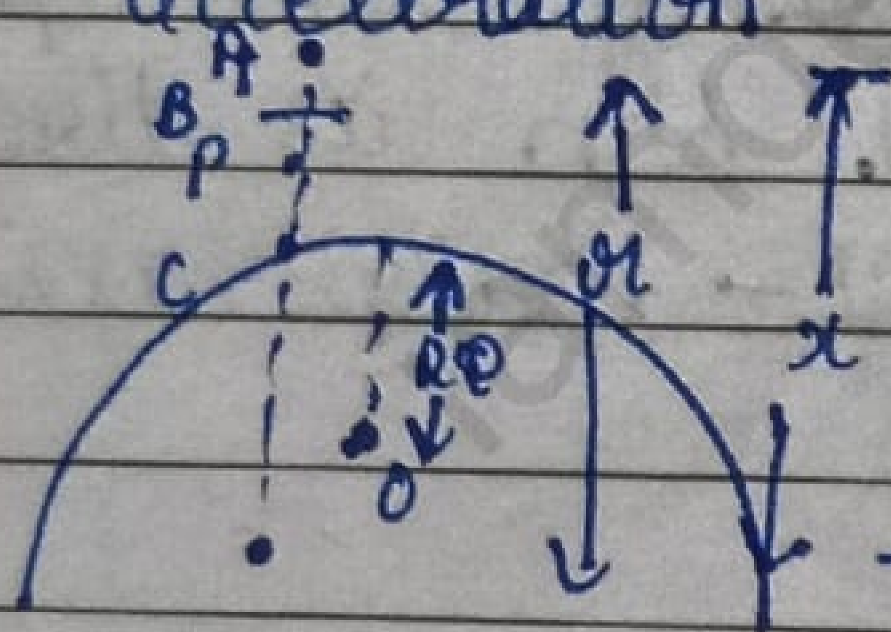
## Gravitational potential energy (G.P.E. or P.E.)

Energy stored in bringing an object of mass m from infinity to any point without change in acceleration is called gravitational P.E.




$$F = \frac{GMem}{x^2} \quad \text{--- (1)}$$

To carry mass 'm' from A to B, required workdone $= dw = Fdx$ --- (2)

$$dw = \frac{GMe}{x^2} m\,dx \quad \text{--- (3)}$$

From $\infty$ to $r$, Required work done

$$\int_0^w dw = \int_\infty^r \frac{GMem}{x^2}\,dx$$

$$w = GMem \int_\infty^r \frac{1}{x^2}\,dx$$

Date / /

$$W = GM_e m\left[-\frac{1}{x}\right]_{\infty}^{r} = -GM_e m\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\boxed{W = \frac{-GM_e m}{r}} \Rightarrow G.PE. \Rightarrow \boxed{V_P \times m}$$

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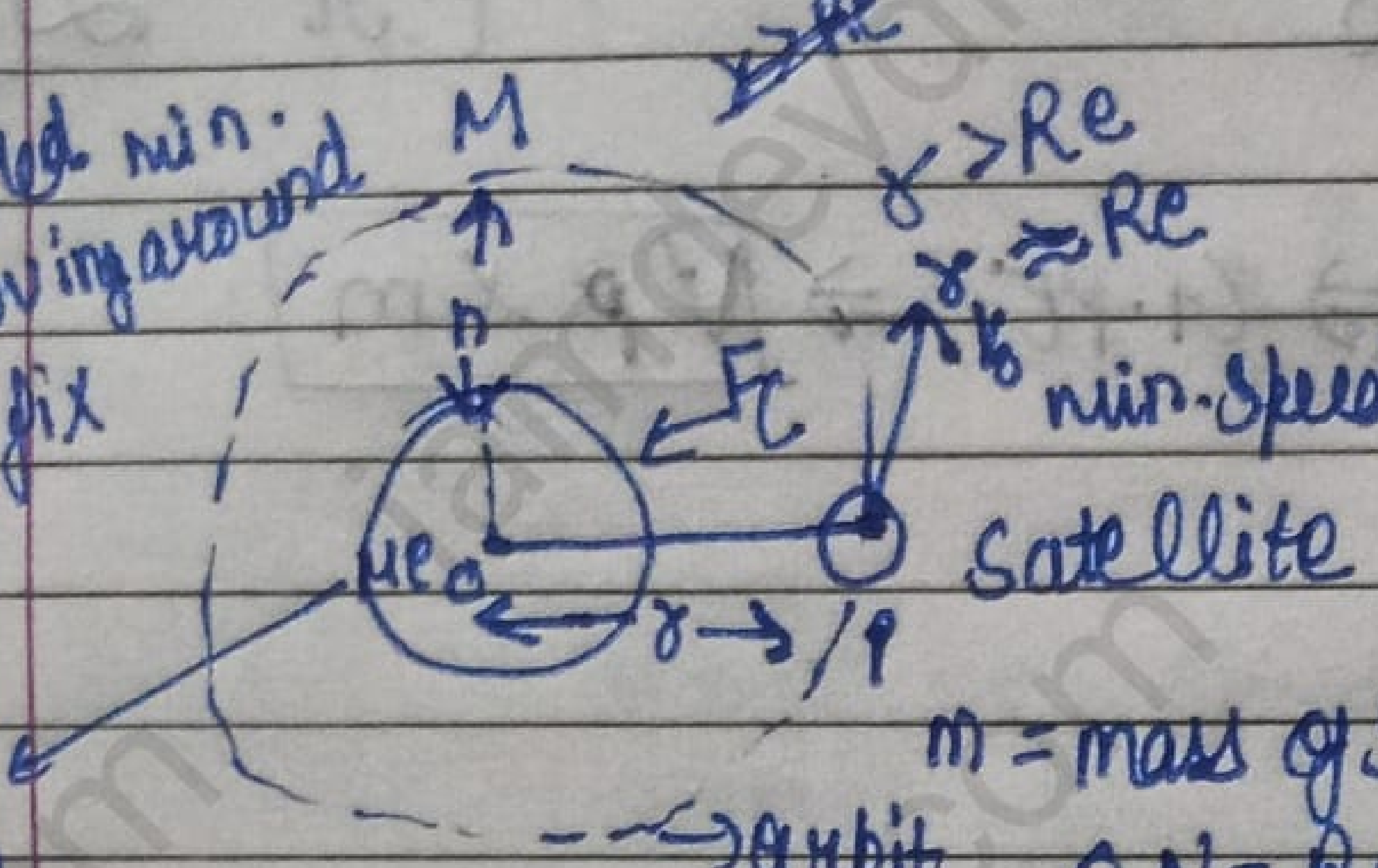
# orbital speed $v_0$

\# Required min. speed moving around earth in fix orbit




m = mass of Satellite

$ON = Re + h$

$OM = OP$

$\boxed{Re + h = r}$

Gravitational force b/w Earth & Satellite

$$F_G = \frac{G Me\, m}{r^2} \quad - (1)$$

$$F_c = \frac{m v_0^2}{r} \quad - (2)$$

for condition of equilibrium

$$F_c = F_G$$

$$\frac{\cancel{m} v_0^2}{\cancel{r}} = G \frac{Me\, \cancel{m}}{r^{\cancel{2}}}$$

$$v_0^2 = \frac{GM}{r}$$

$$v_0 = \sqrt{\frac{Gm}{r}} \quad - (1)$$

$$v_0 = \sqrt{\frac{Gm}{Re + h}} \quad - (2)$$

if $h = 0$

$$v_0 = \sqrt{\frac{Gm}{Re}} \quad - (3)$$

Because we know that

$$g = \frac{G Me}{Re^2}$$

$$G Me = g Re^2$$

$$\frac{G Me}{Re} = g Re \quad - (4)$$

So now from eq (4) & (3)

$$v_0 = \sqrt{g Re} \quad - (5)$$

\# $g = 10 m/s^2$, $Re = 6400 km$

$= 64 \times 10^5$

Now $v_0 = \sqrt{10 \times 64 \times 10^5}$

$= 8 \times 10^3 m/s$

$v_0 = 8 m/s$