Date 17/Jan/2024

Saathi

Unit - X

Oscillations & waves

a) **Oscillations**

1. Periodic motion A motion of a body that repeats itself after regular intervals of time is called periodic motion.

Ex:- The motion of the bob of a simple pendulum about its equilibrium position

Ex:-2. Motion of all planets, satellites, etc around the sun

2. Oscillatory motion:- The motion of a body is said to be oscillated motion if it moved to & fro about a fix point after a regular interval of time. This time motion is also known as vibratory motion

Ex:-1. The motion of bob of simple pendulum about its equilibrium position.

2. The motion of mass attached to its spring about its equilibrium position.

Note:- • Every oscillatory motion of an oscillator is a bounded motion about its mean position.
- It repeats after regular interval of time that is oscillatory motion is required a periodic motion.
- for ex:- Two & fro motion of a pendulum clock is oscillatory as well as periodic motion
- Every periodic motion is not oscillatory motion.
- for ex:- The motion of earth or any other planet around the sun is a periodic motion but not oscillatory.

Displacement in a periodic motion

The distance travel by an oscillatory particle at any instant from its mean position is known as displacement of the particle at that instant.

- The term displacement in a periodic motion represent the change of physical quantity with time.

Difference between Oscillation & Vibration

1. If the frequency of an oscillatory particle or an object is small the particle or object is said to oscillate
for ex:- The oscillation of the branch of tree.
2. If the frequency of an oscillatory particle is large the particle or object is said to be vibrate.
for ex:- The vibration of a string in musical instrument

Periodic function

Any function which repeats itself after a regular interval of time is called periodic function. periodic function are sine & cosine of fixed time period or frequency.

$$f(t) = \sin \omega t$$

$$\omega = \frac{2\pi}{T}$$

$$f(t) = \sin \omega t = \sin \frac{2\pi}{T} t$$

$$g(t) = \cos \omega t = \cos \frac{2\pi}{T} t$$

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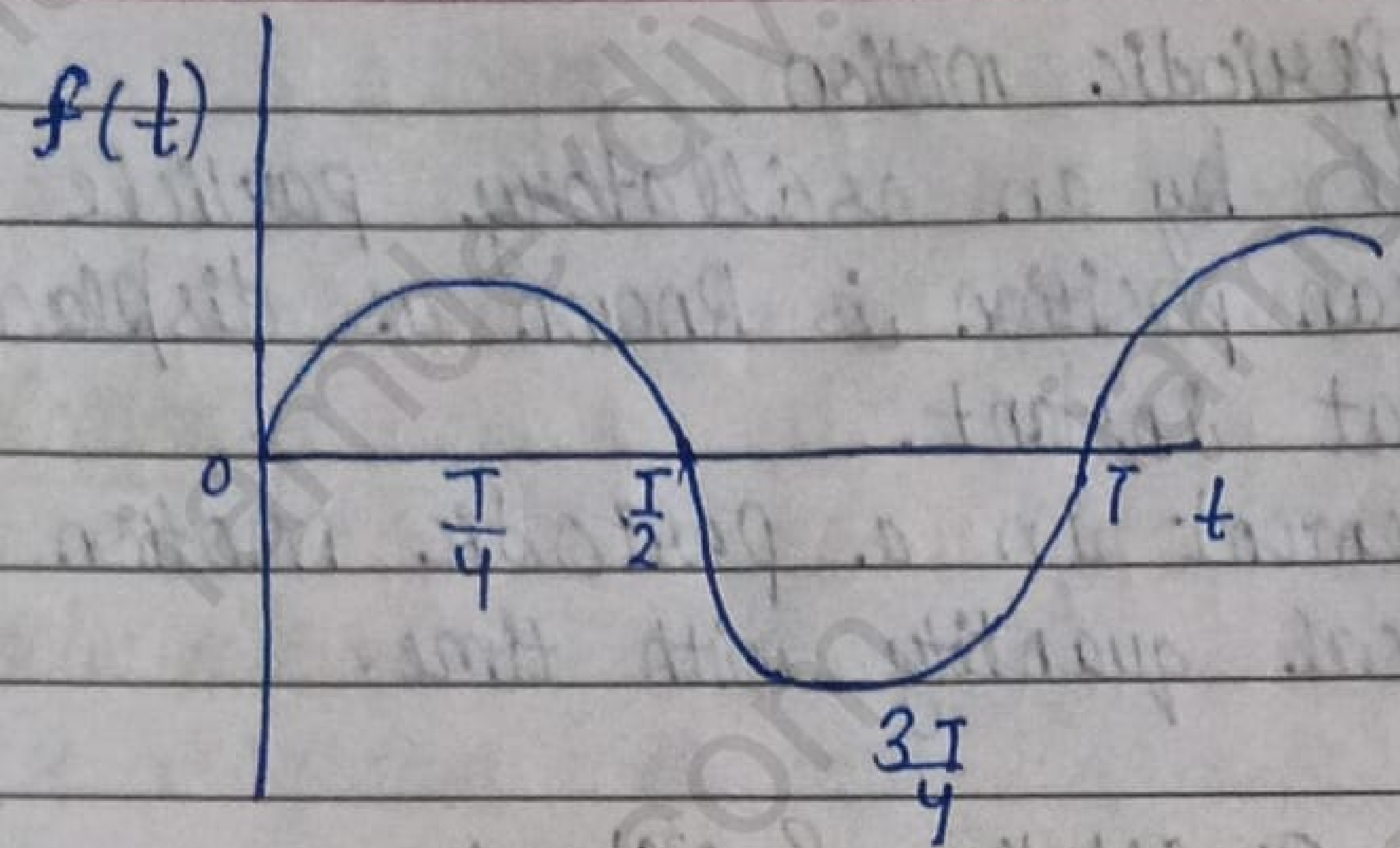
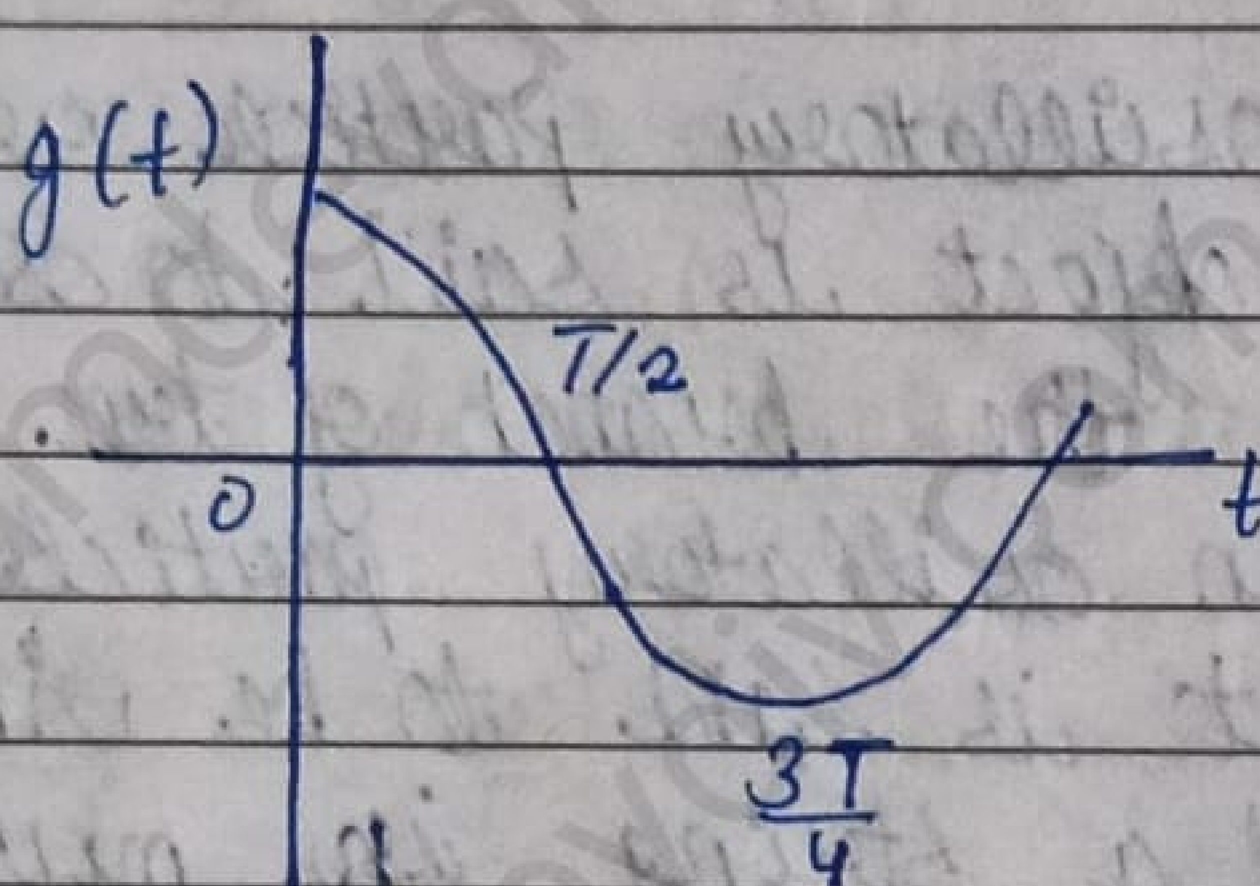
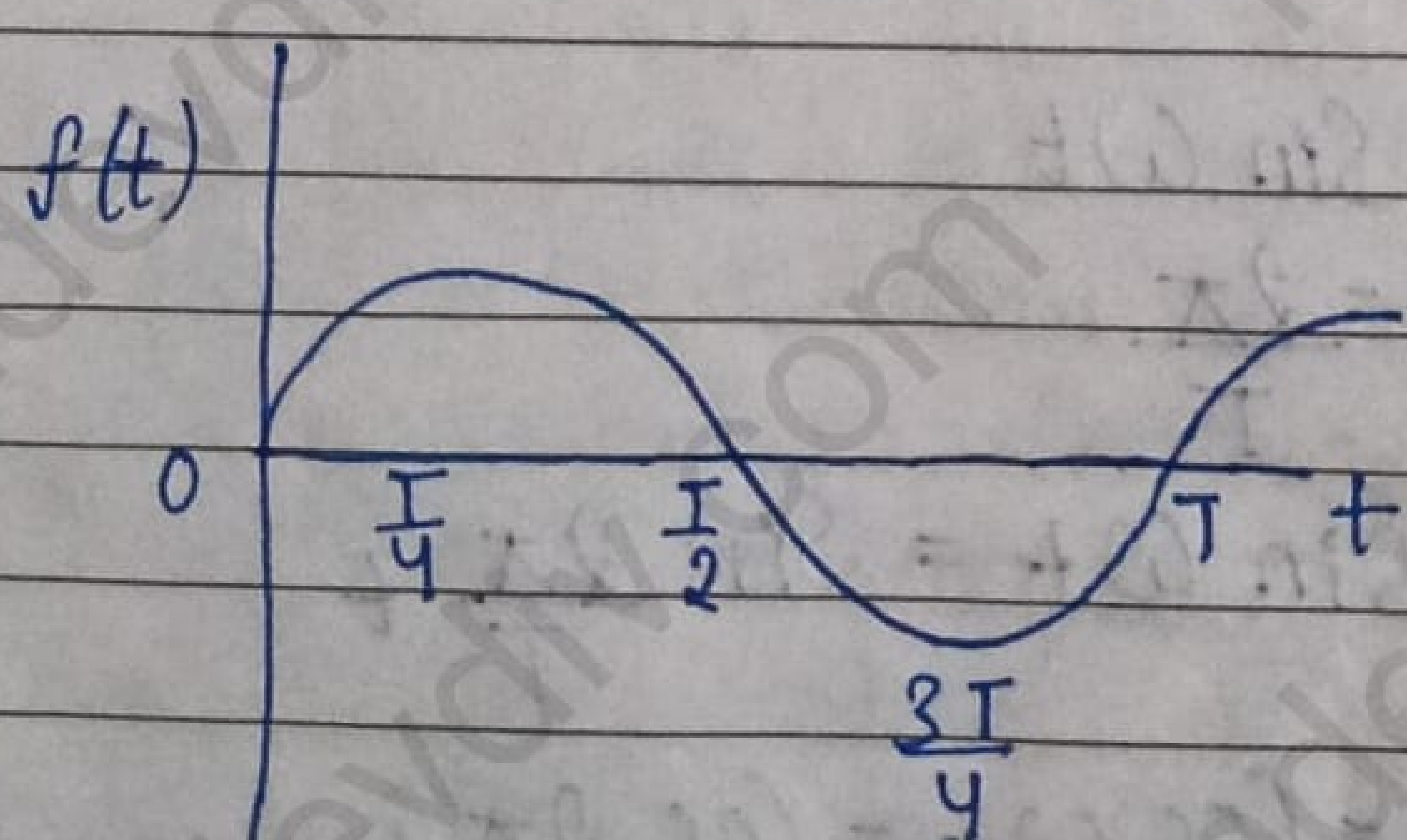




→ <u>Types of periodic function</u>

1. <u>Harmonic function</u>:- The periodic function represent by Cosine and Sine function of period 't' are called Harmonic function. It can be represent by as the following diagram:-



$f(t) = \sin \omega t$

$f(t) = \cos \omega t$

$\omega = \frac{2\pi}{T}$

$f(t) = \sin \frac{2\pi}{T} t$

$f(t) = \cos \frac{2\pi}{T} t$

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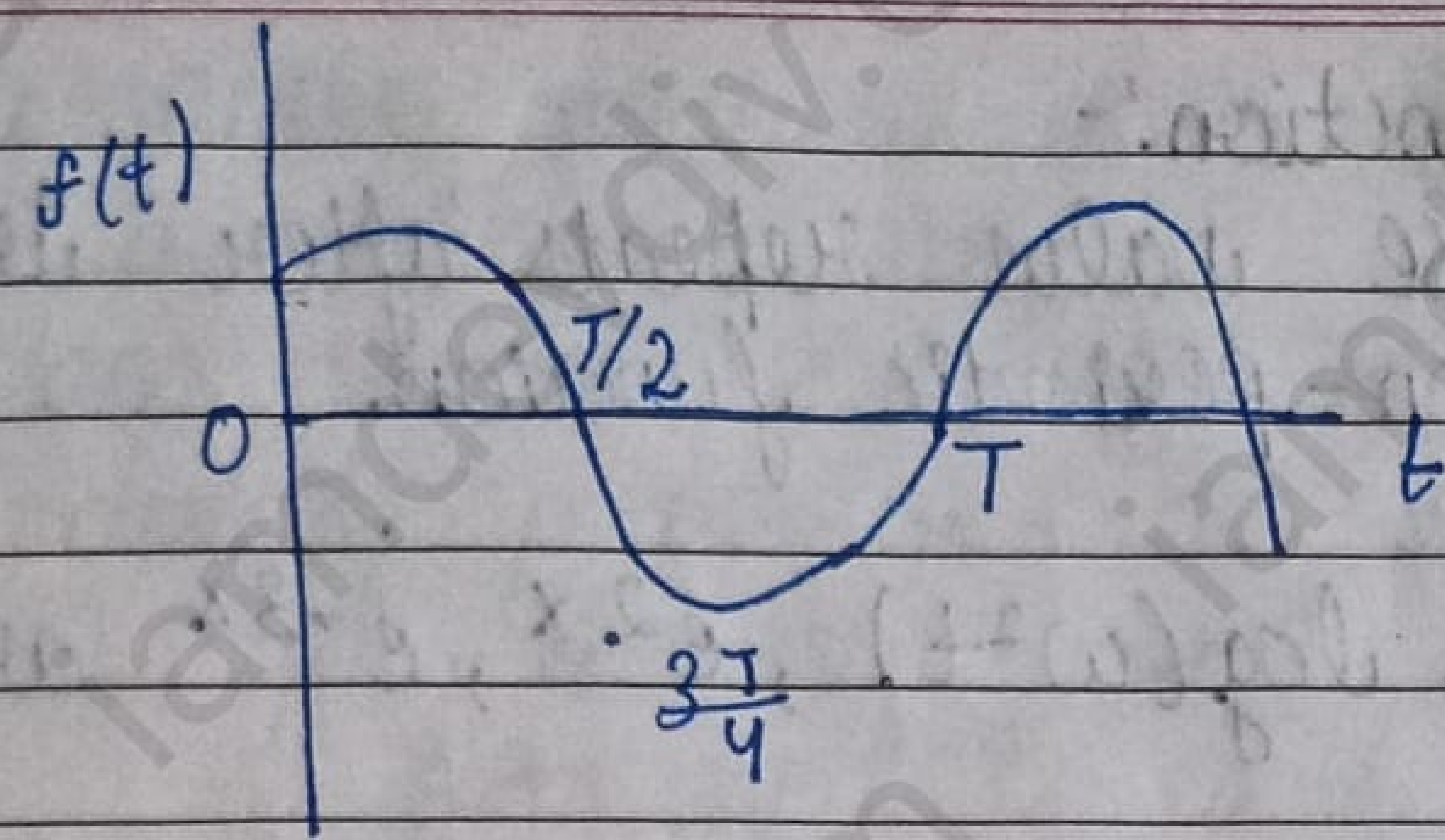
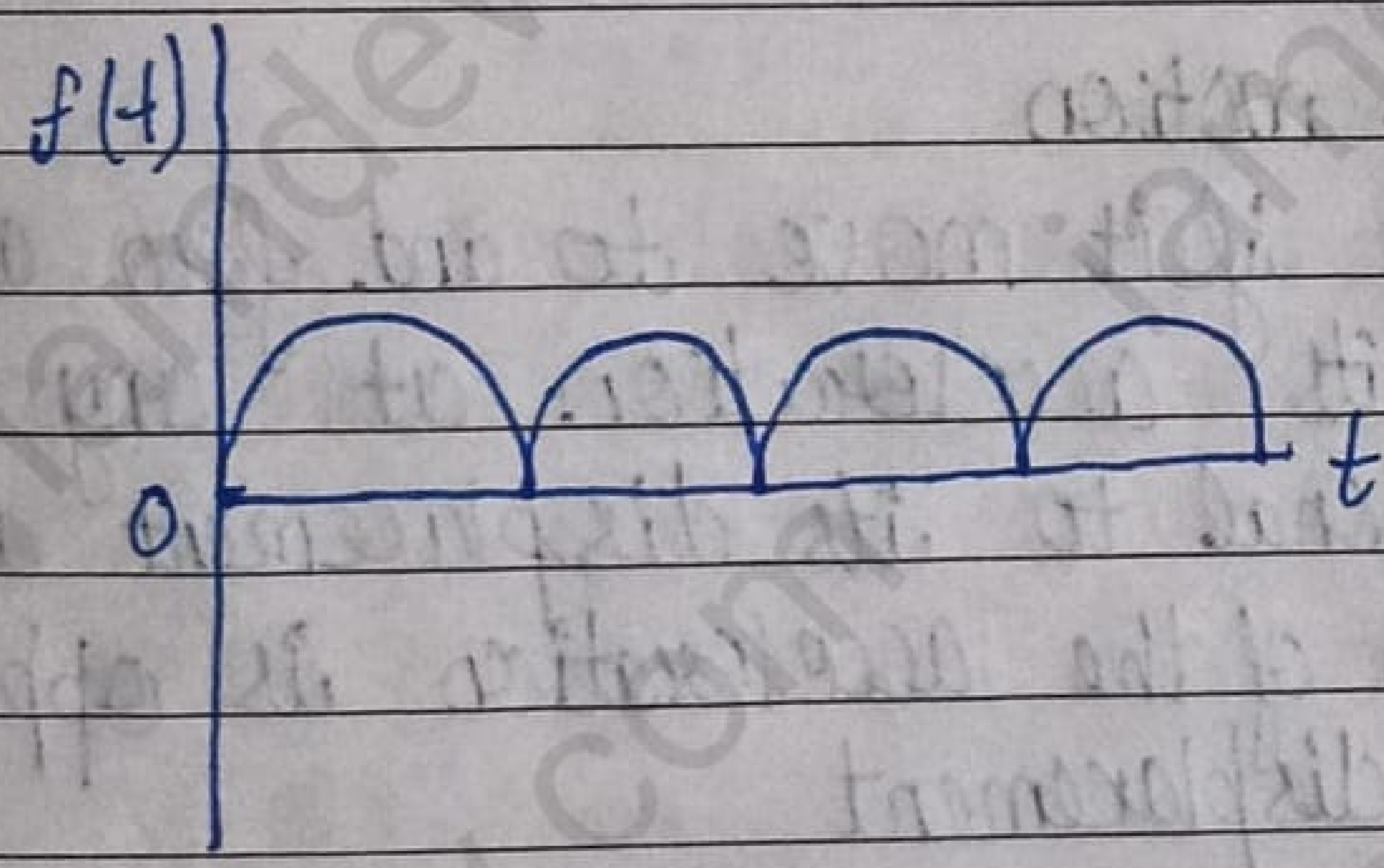
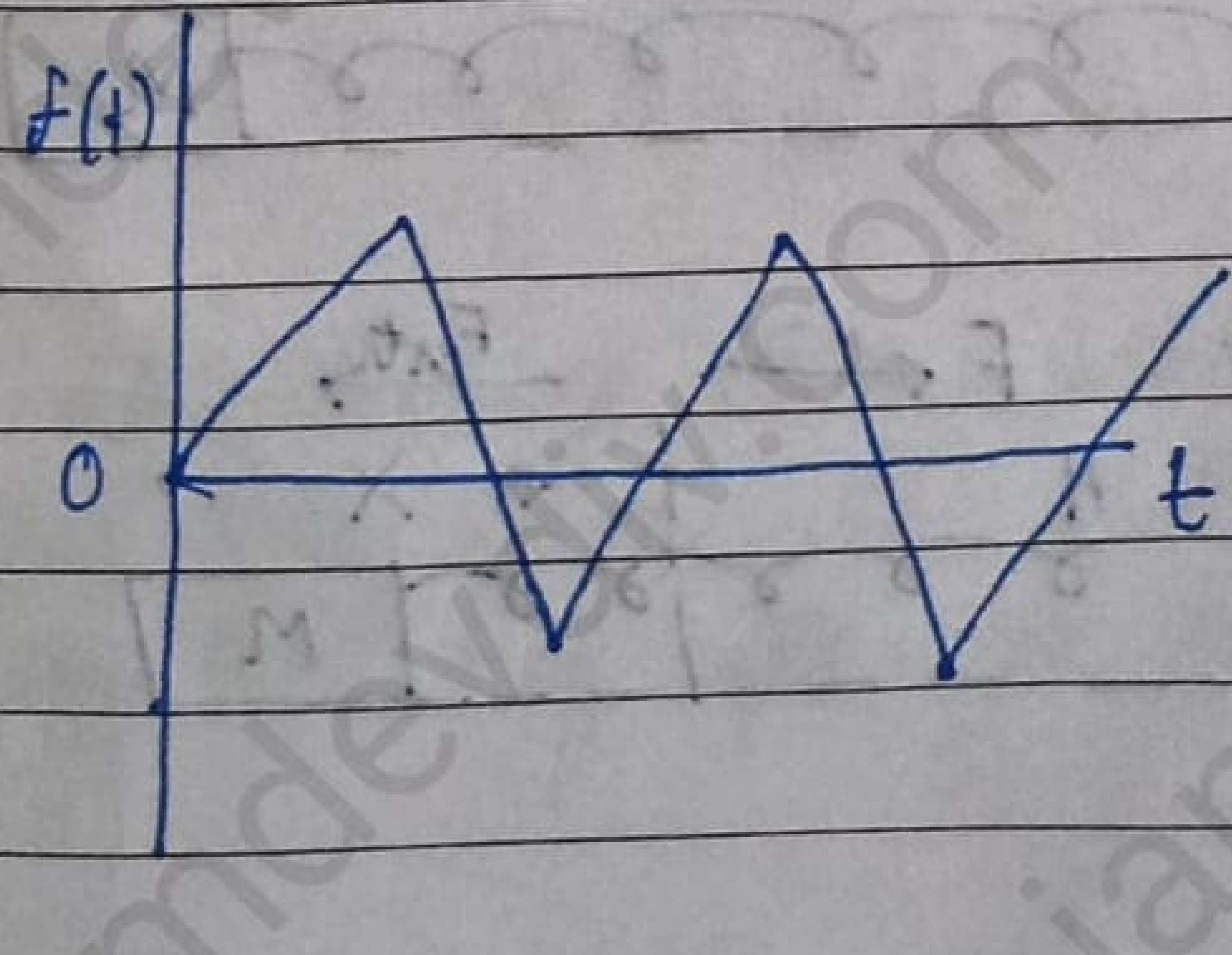


2. Non-harmonic function:-

The periodic function which are not represented by Cosine and Sine function of period 't' are called non-harmonic function.

Non-harmonic function of period 't' as given as





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→ Non-periodic function:-
A function whose value repeats after irregular interval of time is ~~kn~~ non-periodic function.

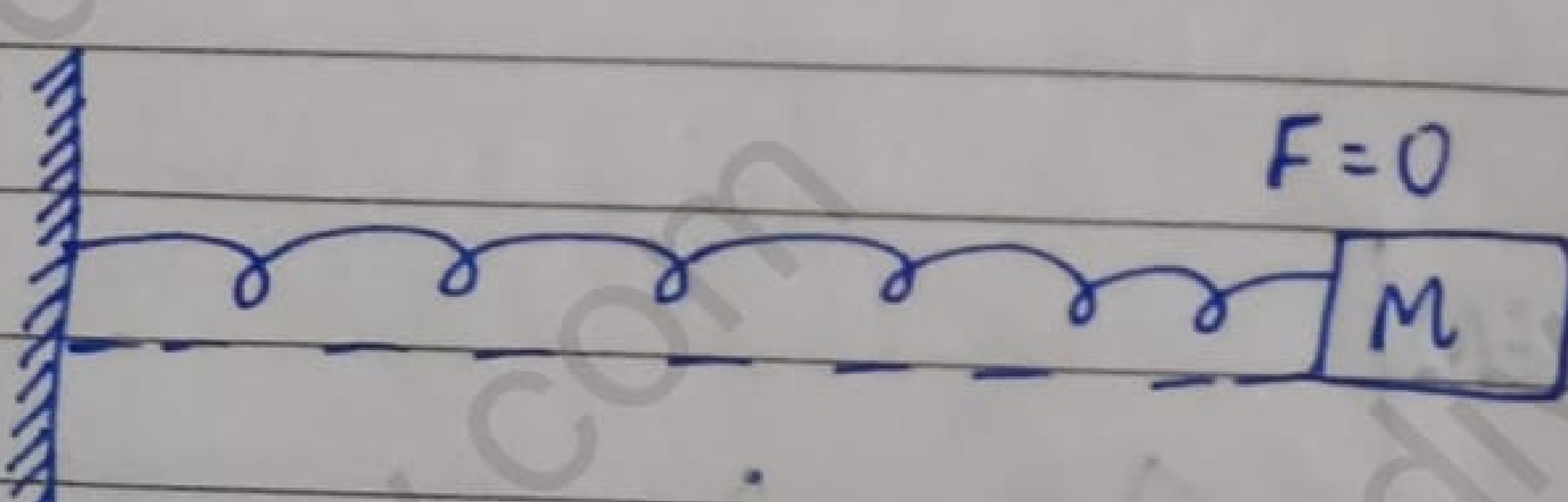
Note:- The $\log(\omega t)$, $\log(\omega^2 t)$, e^{2x}, e^{-3x} are non-periodic function.

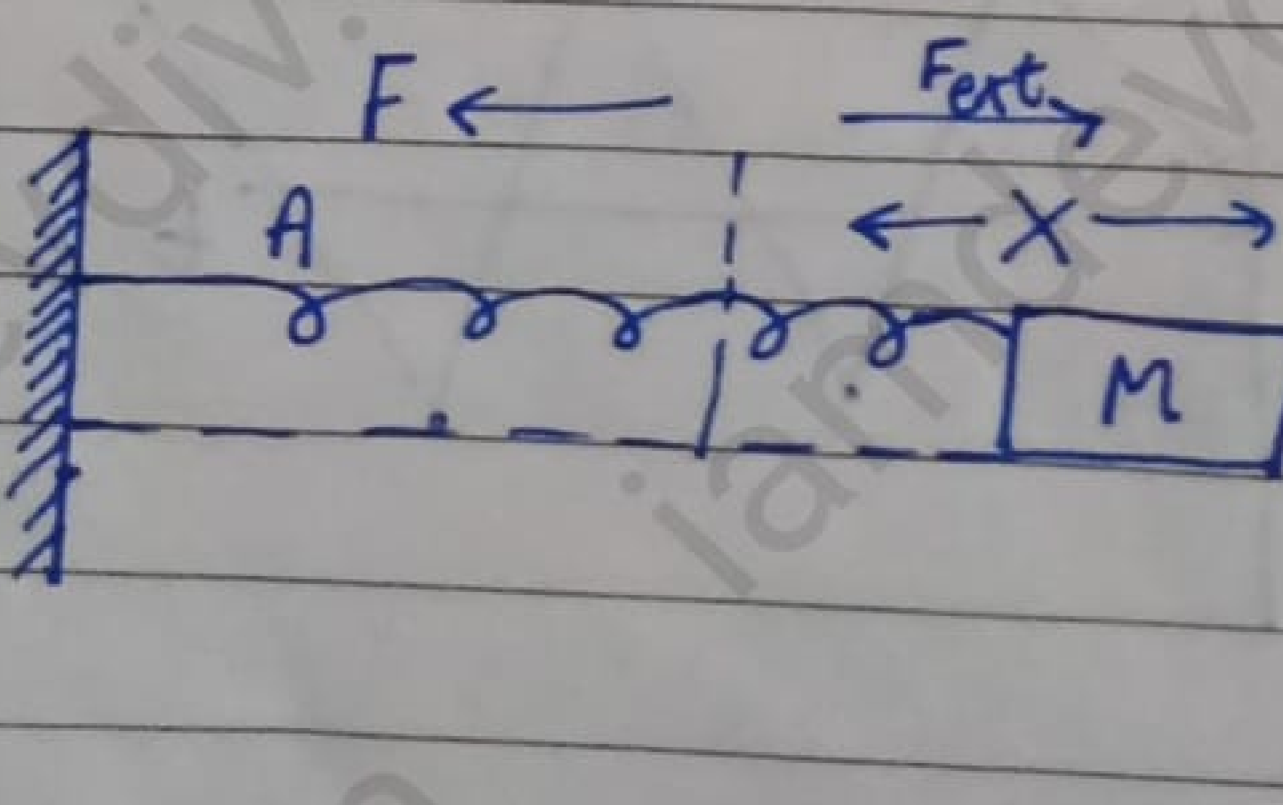
Ques- which of the following function of time represent periodic, non-periodic also give the

(i) $\sin \omega t + \cos \omega t$ - Periodic
(ii) $\sin \omega t + \cos 2\omega t + \sin 4\omega t$ - periodic
(iii) $e^{-\omega t}$ ~~+ Sin 4 6~~ } Non-periodic.
(iv) $\log(\omega t)$

Simple Harmonic motion

A ~~particle~~ particle execute S.H.M. if it move to and fro about a fixed point such that its acceleration at any instant is directly proportional to its displacement at that instant but the direction of the acceleration is opposite to the direction of the displacement





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From the above diagram, it is clear that $\vec{F} = -\vec{F}_{ext}$ — ①
where F is restoring force.
But acc. to Hooke's law. The displacement of the body is directly proportional to the applied ext. force.

$$F_{ext} \propto x \quad \text{— ②}$$

from eqn ① & ②

$$-\vec{F} \propto \vec{x}$$

$$\vec{F} \propto -\vec{x}$$

$$\boxed{\vec{F} = -K\vec{x}} \quad \text{— ③}$$

where K is the constant of proportionality
Or
K is the spring or force constant.

$$K = \left| \frac{\vec{F}}{\vec{x}} \right|$$

Unit of K is N/m.
Dimensional formula of K is $[MT^{-2}]$.
But $F = Ma$ — ④
from ④ & ③

$$-K\vec{x} = Ma$$

$$\boxed{a = -\frac{K\vec{x}}{M}}$$

Note :- $\vec{a} \propto -\vec{x}$

H.W. A spring stressed by 15cm when a body of mass 2.5 kg is suspended from its free end. Calculate the spring constant of spring.

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Note:- 1. Velocity of a particle at any instant executive S.H.M.

$$v = \omega\sqrt{A^2 - y^2}$$

↓ A → Amplitude

y ↳ displacement

2. Acceleration of a particle

$$\frac{d^2y}{dt^2} = a = \omega^2 y$$

3. A general equation for simple harmonic motion can be termed as $F(t) = A\cos(\omega t + \phi)$ → phase difference

$$\omega = \frac{2\pi}{T}$$

Q- A particle executive Simple harmonic motion is represented by a periodic function $F(t) = 5\cos(10t + 0.2)$ then calculate the amplitude angular frequency, frequency, time period & phase difference give disp. represented by $F(t)$ is measured in met. & time in sec.

$A = ?$
$\omega = ?$
$T = ?$
$\eta = ?$
$\phi = ?$

$$F(t) = A\cos(\omega t + \phi)$$
$$F(t) = 5\cos(10t + 0.2)$$

→ $A = 5$

→ $\omega = 10$

$\frac{2\pi}{T} = 10$

→ $\phi = 0.2$

→ $T = \frac{2\pi}{10} = 0.2\pi$

→ $\eta = \frac{1}{T} = \frac{1}{\frac{2\pi}{10}} = \frac{10}{2\pi} = \frac{5}{\pi}$

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~~Only formula~~

Ques- Oscillation of mass attached with a vertical string

The Time period of oscillation of mass 'M' is given by

$$T = 2\pi\sqrt{\frac{M}{K}}$$ M → Mass, K → Spring Constant

Ques- The period of oscillation of a body of mass M ~~to a~~ is suspended to a string is 2 sec. What will be the period of oscillation. If a body of mass 4M is attached to the same spring

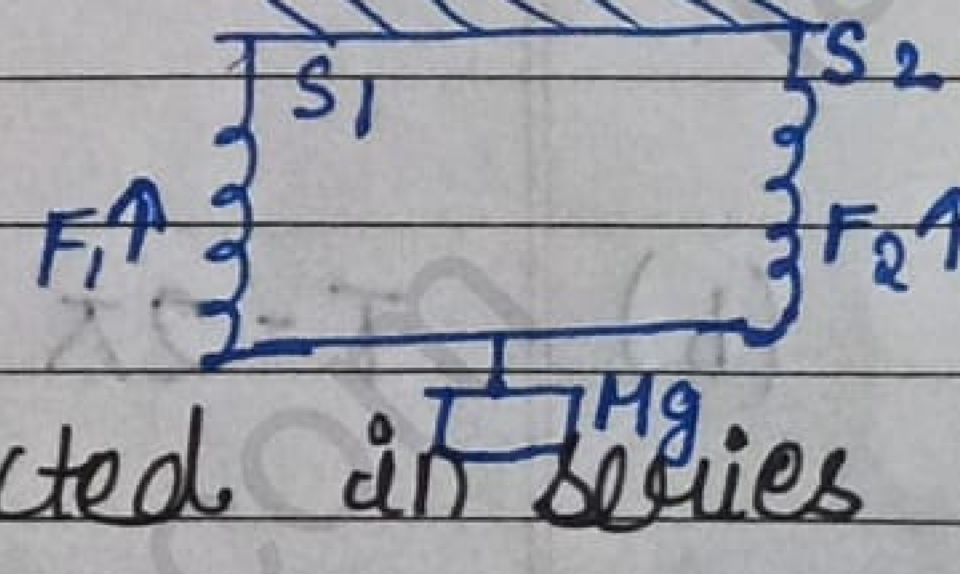
→

★ Only formula

Motion of a body suspended by two spring connected in parallel

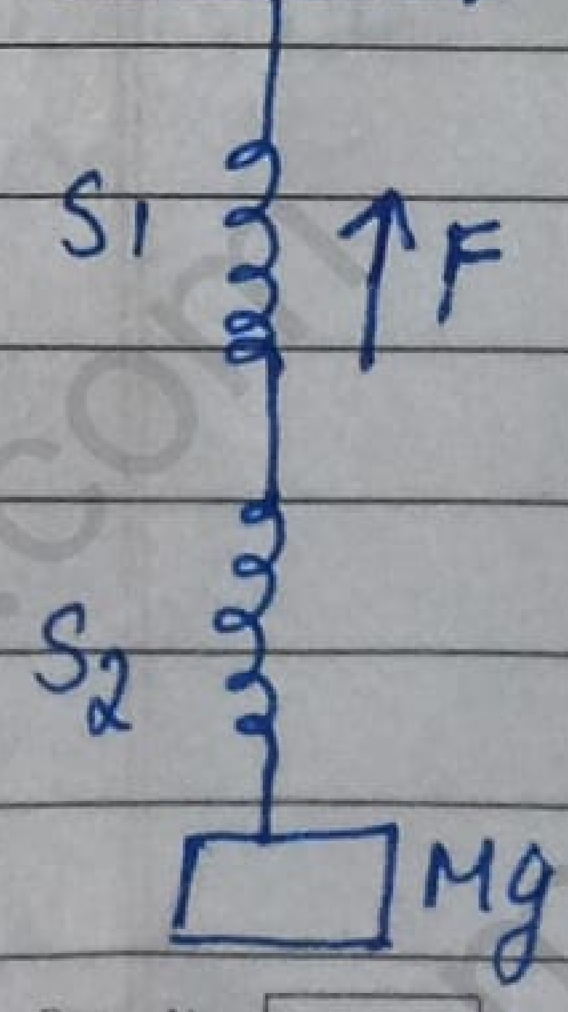
The time period of the oscillated body in this case

$$T = 2\pi\sqrt{\frac{M}{K_1+K_2}} \quad \text{OR} \quad T = 2\pi\frac{M}{2K}$$



Motion of a body by two spring connected in series

Time period of the oscillated body in this case



$$T = 2\pi\sqrt{\frac{(K_1+K_2)M}{K_1K_2}}$$

$$\text{OR } T = 2\pi\sqrt{\frac{2M}{K}} \quad (\because K_1 = K_2 = K)$$

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Q- A body of mass 2 kg is suspended by two identical spring connected in parallel with a rigid body.
(a) Find the time period of osillation of the body if it is displaced from its mean position & then released
(b) Now the body is suspended by same spring connected in series with rigid body Find the period of osillation in this case also

$K = 196\ N/m$

$M = 2\ kg$

~~$T = 2\pi\sqrt{\frac{2M}{K}}$~~

~~$T = 2\pi\ \frac{2\times 1962}{196}$~~

(a) $T = 2\pi\sqrt{\frac{M}{2K}}$

$T = 2\pi\sqrt{\frac{2}{196\times 2}} \Rightarrow T = 2\times\frac{22}{7}\times\frac{1}{14_7}$

$T = \frac{22}{49}$

(b) $T = 2\pi\sqrt{\frac{2M}{K}}$

$T = 2\pi\sqrt{\frac{2\times 2}{196}}$

$T = 2\times\frac{22}{7}\times\frac{2}{14_7}$

$T = \frac{44}{49}$ (Ans)