12/4/23

CHAPTER-3
MATRICES

★ MATRIX

A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

$$\underset{\text{denoted by capital letter}}{A} = \begin{bmatrix} -2 & 5 \\ 0 & \sqrt{5} \\ 3 & 6 \end{bmatrix}_{3\times 2}$$

• ORDER OF A MATRIX

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1j} \cdots a_{1n} \\ a_{21} & a_{22} & a_{23} \cdots a_{2j} \cdots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & a_{i3} \cdots a_{ij} \cdots a_{in} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} \cdots a_{mj} \cdots a_{mn} \end{bmatrix}_{m\times n} \leftarrow \text{order of matrix}$$

$A = [a_{ij}]_{m\times n}$

$1 \le i \le m \rightarrow$ rows

$1 \le j \le n \rightarrow$ columns

$i, j \in N$

• TYPES OF MATRICES

(i) Column matrix

has only one column

$$A = \begin{bmatrix} 0 \\ \sqrt{3} \\ -1 \end{bmatrix}_{3\times 1}$$

(ii) Row matrix

has only one row.

$$B = \begin{bmatrix} \sqrt{5} & 2 & 3 \end{bmatrix}_{1\times 3}$$

(iii) Zero/null matrix

All elements are zero

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

(iv) Square matrix

Number of rows = Number of columns

$$A = \begin{bmatrix} 3 & -1 & 4 \\ 4 & 6 & 7 \\ 5 & 9 & 3 \end{bmatrix}_{3\times 3}$$

these are not diagonal elements

diagonal elements

(v) Diagonal matrix

- All non-diagonal elements are zero.

Diagonal matrix is a square matrix.

$$A = \begin{bmatrix} -1.1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$$

off diagonal

main diagonal

(vi) Scalar matrix

Diagonal elements are equal and rest are zero.

Scalar matrix is a square matrix and a diagonal matrix.

$$A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}_{3\times 3}$$

(vii) Identity matrix

All diagonal elements are 1 and rest are zero.

Identity matrix is a square matrix, a diagonal matrix and a scalar matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) they are of the same order.

(ii) each element of A is equal to the corresponding element of B.

$$\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \longrightarrow \text{Equal matrices}$$

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \longrightarrow \text{Not equal matrices}$$

12/4/23

☆ EXERCISE - 3.1

Q1.) In the matrix, $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write

(i) The order of the matrix,

(ii) The number of elements,

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

Sol. (i) 3×4

(ii) 12

(iii) $a_{13} = 19$

$a_{21} = 35$

$a_{33} = -5$

$a_{24} = 12$

$a_{23} = \frac{5}{2}$

Q2.) If a matrix has 24 . . . . 13 elements?

Sol. For 24 elements,

possible order pairs $= \underset{1\times24}{(1,24)}, \underset{24\times1}{(24,1)}, \underset{2\times12}{(2,12)}, \underset{12\times2}{(12,2)}, \underset{3\times8}{(3,8)}, \underset{8\times3}{(8,3)}, \underset{4\times6}{(4,6)}, \underset{6\times4}{(6,4)}$

For 13 elements,

possible order pairs $= \underset{1\times13}{(1,13)}, \underset{13\times1}{(13,1)}$

Q3.) If a matrix has 18 . . . . 5 elements?

Sol. For 18 elements,

possible order pairs $= \underset{1\times18}{(1,18)}, \underset{18\times1}{(18,1)}, \underset{2\times9}{(2,9)}, \underset{9\times2}{(9,2)}, \underset{3\times6}{(3,6)}, \underset{6\times3}{(6,3)}$

For 5 elements,

possible orders $\Rightarrow (1,5), (5,1)$

$1\times 5 \quad 5\times 1$

Qu.) Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by :

(i) $a_{ij} = \dfrac{(i+j)^2}{2}$

(ii) $a_{ij} = \dfrac{i}{j}$

(i) $i = 1, 2$
$j = 1, 2$

$a_{11} = \dfrac{(1+1)^2}{2} = \dfrac{4}{2} = 2$

$a_{12} = \dfrac{(1+2)^2}{2} = \dfrac{9}{2}$

$a_{21} = \dfrac{(2+1)^2}{2} = \dfrac{9}{2}$

$a_{22} = \dfrac{(2+2)^2}{2} = 8$

$$\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

(ii) $i = 1, 2$
$j = 1, 2$

$a_{11} = \dfrac{1}{1} = 1$

$a_{12} = \dfrac{1}{2}$

$a_{21} = \dfrac{2}{1} = 2$

$a_{22} = \dfrac{2}{2} = 1$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

(iii) $a_{ij} = \dfrac{(i+2j)^2}{2}$

(iii) $i = 1, 2$
$j = 1, 2$

$a_{11} = \dfrac{(1+2(1))^2}{2} = \dfrac{9}{2}$

$a_{12} = \dfrac{(1+2(2))^2}{2} = \dfrac{25}{2}$

$$a_{21} = \frac{(2+2(1))^2}{2} = 8$$

$$a_{22} = \frac{(2+2(2))^2}{2} = 18$$

$$\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

(Q5.) Construct a 3×4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i+j|$

(i) $i = 1,2,3$
$j = 1,2,3,4$

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

$a_{11} = \frac{1}{2}|-3(1)+1| = 1$

$a_{12} = \frac{1}{2}|-3(1)+2| = +\frac{1}{2}$

$a_{13} = \frac{1}{2}|-3(1)+3| = 0$

$a_{14} = \frac{1}{2}|-3(1)+4| = \frac{1}{2}$

$a_{21} = \frac{1}{2}|-3(2)+1| = +\frac{5}{2}$

$a_{22} = \frac{1}{2}|-3(2)+2| = +2$

$a_{23} = \frac{1}{2}|-3(2)+3| = +\frac{3}{2}$

$a_{24} = \frac{1}{2}|-3(2)+4| = +1$

$a_{31} = \frac{1}{2}|-3(3)+1| = +4$

$a_{32} = \frac{1}{2}|-3(3)+2| = +\frac{7}{2}$

$a_{33} = \frac{1}{2}|-3(3)+3| = +3$

$a_{34} = \frac{1}{2}|-3(3)+4| = +\frac{5}{2}$

(ii) $a_{ij} = 2i-j$

(ii) $i = 1,2,3 \quad j = 1,2,3,4$

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

$a_{11} = 2(1)-1 = 1$

$a_{12} = 2(1)-2 = 0$

$a_{13} = 2(1)-3 = -1$

$a_{14} = 2(1)-4 = -2$

$a_{21} = 2(2)-1 = 3$

$a_{22} = 2(2)-2 = 2$

$a_{23} = 2(2)-3 = 1$

$a_{24} = 2(2)-4 = 0$

$a_{31} = 2(3)-1 = 5$

$a_{32} = 2(3)-2 = 4$

$a_{33} = 2(3)-3 = 3$

$a_{34} = 2(3)-4 = 2$

13/4/23

☆ OPERATIONS ON MATRICES

• ADDITION

A and B are two matrices of order m x n.

$$A + B = C$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}_{2\times3}$$

$$B = \begin{bmatrix} 3 & 9 & 6 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 9 & 6 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 12 & 10 \\ 9 & 8 & 8 \end{bmatrix}_{2\times3}$$

* PROPERTIES

(i) Commutative

$$A + B = B + A$$

(ii) Associative

$$(A+B) + C = A + (B+C)$$

(iii) Additive identity

$$A + O = A = O + A$$

(iv) Additive inverse

$$A + \underset{\substack{\uparrow \\ \text{Additive inverse}}}{(-A)} = O = (-A) + A$$

• SUBTRACTION

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 2 & 4 \\ 3 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 0 & 5 \end{bmatrix}$$

- MULTIPLICATION

$A = [a_{ij}]_{m \times n}$

$B = [b_{ij}]_{n \times p}$

AB is possible, BA is not possible

No. of columns of pre multiplier = No. of rows of post multiplier

$$A = [\ ]_{m \times n} \times B = [\ ]_{n \times p} \Rightarrow AB = [\ ]_{m \times p}$$

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 9 \end{bmatrix}_{2 \times 2}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$$

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 2 \times 1 + 4 \times 4 & 2 \times 2 + 4 \times 5 & 2 \times 3 + 4 \times 6 \\ 3 \times 1 + 9 \times 4 & 3 \times 2 + 9 \times 5 & 3 \times 3 + 9 \times 6 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 2 + 16 & 4 + 20 & 6 + 24 \\ 3 + 36 & 6 + 45 & 9 + 54 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 18 & 24 & 30 \\ 39 & 51 & 63 \end{bmatrix}$$

14|4|23

★ EXERCISE - 3.1 (CONTINUE)

Q6.) Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(i) $y = 4$
$x = 1$
$z = 3$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(ii) $5+z = 5 \Rightarrow \boxed{z = 0}$
$x + y = 6$ —— ①
$xy = 8$ —— ②

$\Rightarrow \cancel{(x+y)^2} = \cancel{36}$ $\quad \cancel{(x+y)^2 = (x-y)^2 + 4xy}$
$\Rightarrow \cancel{x^2 + y^2 + 2xy = 36} \Rightarrow \cancel{x^2+y^2+2xy} = \cancel{x^2+y^2-2xy+4xy}$
$\Rightarrow \cancel{x^2+y^2+}$

$(x+y)^2 = (x-y)^2 + 4xy$
$\Rightarrow 36 = (x-y)^2 + 32$
$\Rightarrow (x-y)^2 = 4$
$\Rightarrow x - y = \pm 2$

(+2)

$x + y = 6$
$+ \; x - y = 2$
$2x = 8$
$\boxed{x = 4}$
$\boxed{y = 2}$

(−2)

$x + y = 6$
$+ \; x - y = -2$
$2x = 4$
$\boxed{x = 2}$
$\boxed{y = 4}$

Q7.) Find the value of a, b, c and from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Sol. $a-b=-1$ — ①

$2a-b=0$ — ②

$2a+c=5$ — ③

$3c+d=13$ — ④

② − ①

$2a-\cancel{b}=0$

$a-\cancel{b}=-1$

$- \quad + \quad +$

$a = 1$

② ⇒ $2(1)-b=0$

⇒ $b=2$

③ ⇒ $2(1)+c=5$

⇒ $c=3$

④ ⇒ $3(3)+d=13$

⇒ $d=4$

$a=1$, $b=2$, $c=3$, $d=4$ Ans

Q8.) $A=[a_{ij}]_{m\times n}$ is a square matrix, if

(A) $m<n$

(B) $m>n$

(C) $m=n$

(D) None of these

Q9.) Which of the given values of x and y make the following pair of matrices equal $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$, $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

(A) $x = \frac{-1}{3}$, $y = 7$ (B) Not possible to find

(C) $y = 7$, $x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}$, $y = \frac{-2}{3}$

Sol. $3x+7 = 0 \Rightarrow x = \frac{-7}{3}$
$y-2 = 5 \Rightarrow y = 7$
$y+1 = 8 \Rightarrow y = 7$
$2-3x = 4 \Rightarrow x = \frac{-2}{3}$

Contradictory

$\therefore$ (B) Not possible to find

(Q10.) The number of all possible matrices of order 3×3 with each entry 0 or 1 is: (A) 27 (B) 18 (C) 81 (D) 512

Sol. ~~Every~~ All $3 \times 3 = 9$ places can be filled with two possible values (0 or 1)
$\therefore 2^9 =$ (D) 512 Ans

15/4/23

★ ~~SCA~~ OPERATIONS ON MATRICES (CONTINUE)

- SCALAR MULTIPLICATION

K is ~~we~~ any non-zero scalar, A is a matrix
KA is a matrix obtained by multiplying each element of A by K

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 12 \\ 15 & 18 & 21 \end{bmatrix}$$

* PROPERTIES

(i) Matrix Mutiplication is not Commutative

$$AB \neq BA$$

$$AB = BA \iff A = B$$

$$(A+B)^2 = (A+B)(A+B)$$
$$= A^2 + AB + BA + B^2$$

$$(A+B)^3 = (A+B)^2(A+B)$$
$$= (A^2 + AB + BA + B^2)(A+B)$$
$$= A^3 + A^2B + ABA + AB^2 + BA^2 + BAB + B^2A + B^3$$

(ii) Associative

$$(AB)C = A(BC)$$

(iii) Multiplicative Identity

$$AI = A = IA$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

(iv) Multiplicative Inverse

If $AB = I = BA$,

$A^{-1} = B$

A is called invertible

(v) k and l are non-zero scalars

$(k+l)A = kA + lA$

$k(A+B) = kA + kB$

$k(A-B) = kA - kB$

$(kl)A = k(lA) = l(kA)$

15/4/23

☆ EXERCISE - 3.2

Q1.) Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following:

(i) A + B

(i) $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(ii) A - B

(ii) $A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(iii) 3A - C

(iii) $3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(iv) AB

(iv) $AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

(v) BA

(v) $BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

Q2.) Compute the following:

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2 \cancel{2ac} + b^2 - 2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(i) $\begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iii) $\begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q4.) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A + (B-C) = (A+B) - C$.

Sol. $A + B = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$

$B - C = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

$A + (B-C) = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} = (A+B) - C = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$

Verified

Q3) Compute the indicated products.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

(i) $\begin{bmatrix} a^2+b^2 & \cancel{-ab+ab}\ 0 \\ 0 & a^2+b^2 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

(ii) $\cancel{[1\times2+2\times3+3\times4]} = \cancel{[20]}$ $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

(iv) $\begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & \overset{15+16+25}{\cancel{20+20+30}} \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

(v) $\begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 6-1+9 & -9+0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

Q5.) If $A = \begin{bmatrix} 2/3 & 1 & 5/3 \\ 1/3 & 2/3 & 4/3 \\ 7/3 & 2 & 2/3 \end{bmatrix}$ and $B = \begin{bmatrix} 2/5 & 3/5 & 1 \\ 1/5 & 2/5 & 4/5 \\ 7/5 & 6/5 & 2/5 \end{bmatrix}$, then compute $3A - 5B$.

Sol. $3A - 5B = 3\begin{bmatrix} 2/3 & 1 & 5/3 \\ 1/3 & 2/3 & 4/3 \\ 7/3 & 2 & 2/3 \end{bmatrix} - 5\begin{bmatrix} 2/5 & 3/5 & 1 \\ 1/5 & 2/5 & 4/5 \\ 7/5 & 6/5 & 2/5 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Q6.) Simplify $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Sol. $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q7) Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(i) $X + Y + X - Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow X + X + Y - Y = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow 2X + 0 = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\Rightarrow \boxed{X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}}$

$X + Y - (X - Y) = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow X + Y - X + Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$\Rightarrow X - X + Y + Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$\Rightarrow 0 + 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$\Rightarrow Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$

$\Rightarrow \boxed{Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$.

(ii) $2X + 3Y + 3X + 2Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

$$\Rightarrow 2X + 3X + 3Y + 2Y = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$\Rightarrow 5X + 5Y = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$\Rightarrow 5(X+Y) = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$\Rightarrow X + Y = \frac{1}{5}\begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$\Rightarrow X + Y = \begin{bmatrix} 4/5 & 1/5 \\ 3/5 & 1 \end{bmatrix}$$

$$3X + 2Y - (2X + 3Y) = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3X + 2Y - 2X - 3Y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\Rightarrow 3X - 2X + 2Y - 3Y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\Rightarrow X - Y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$X + Y + X - Y = \begin{bmatrix} 4/5 & 1/5 \\ 3/5 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\frac{1}{5} - \frac{5}{1}$$
$$\frac{1-25}{5} = \frac{-24}{5}$$

$$\Rightarrow X + X + Y - Y = \begin{bmatrix} 4/5 & -24/5 \\ -22/5 & 6 \end{bmatrix}$$

$$\Rightarrow 2X + 0 = \begin{bmatrix} 4/5 & -24/5 \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} 4/5 & -24/5 \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow \boxed{X = \begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}}$$

$$-X + X + Y = \begin{bmatrix} 4/5 & 1/5 \\ 3/5 & 1 \end{bmatrix} - X$$

$$\Rightarrow 0 + Y = \begin{bmatrix} 4/5 & 1/5 \\ 3/5 & 1 \end{bmatrix} - \begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}$$

$$\Rightarrow \boxed{Y = \begin{bmatrix} 2/5 & 13/5 \\ 14/5 & -2 \end{bmatrix}}$$

Q8.) Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

Sol. $2X + Y - Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - Y$

$$\Rightarrow 2X + 0 = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$ Ans

---

Q9.) Find x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Sol. $\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$2 + y = 5 \Rightarrow y = 3$ Ans

$2x + 2 = 8 \Rightarrow x = 3$

~~Q10.) Solve th~~

Q 10.) Solve the equation for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Sol. $\begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$$\left.\begin{aligned} 2x+3 = 9 &\Rightarrow x = 3 \\ 2z-3 = 15 &\Rightarrow z = 9 \\ 2y = 12 &\Rightarrow y = 6 \\ 2t+6 = 18 &\Rightarrow t = 6 \end{aligned}\right\} \text{Ans}$$

Q11.) If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, Find the values of x and y.

Sol. $\begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$2x - \cancel{y} = 10$

$3x + \cancel{y} = 5$

$5x = 15$

$x = 3$

$2(3) - y = 10$

$\Rightarrow 6 - y = 10$

$\Rightarrow y = -4$

$x = 3, y = -4$ Ans

Q12.) Given $3\begin{bmatrix} x & y \\ 2 & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w.

Sol. $\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & x+y+6 \\ z+w-1 & 2w+3 \end{bmatrix}$

$3x = x+4 \Rightarrow x = 2$

$3y = x+y+6 \Rightarrow y = \cancel{\frac{7}{2}}\ 4$

$3w = 2w+3 \Rightarrow w = 3$

$3z = z+w-1 \Rightarrow z = 1$

Ans

Q13.) IF $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$.

Sol. $F(x)F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\cancel{\Rightarrow F(x)F(y) = \begin{bmatrix} \cos x\cos y & \sin x\sin y & 0 \\ \sin x\sin y & \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}}$

$\Rightarrow F(x)F(y) = \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0+0+0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$

$\Rightarrow \underset{(L.H.S.)}{F(x)F(y)} = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\underset{(R.H.S)}{F(x+y)} = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

L.H.S. = R.H.S.

Hence, ~~proved~~ verified.

Q14.) Show that

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(i) $\begin{bmatrix} 10+(-3) & 5+(-4) \\ 12+21 & 6+28 \end{bmatrix} \neq \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \neq \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Hence, verified

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} -1+0+6 & 1+(-2)+9 & 0+2+12 \\ 0+0+0 & 0+(-1)+0 & 0+1+0 \\ -1+0+0 & 1+(-1)+0 & 0+1+0 \end{bmatrix} \neq \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ 0+0+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \neq \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

Hence, verified

Q15.) Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

Sol. $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0+(-1)+0 & 1+(-3)+0 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$ Ans

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Q16) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

Sol. $$A^2 = A.A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$A^3 - 6A^2 + 7A + 2I$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 21-30+7+2 & 0-0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0-0+0+0 & 55-78+21+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow 0$$

hence, proved

Q17.) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

Sol. $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+(-8) & -6+4 \\ 12+(-8) & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$3k-2 = 1 \Rightarrow k = 1$

$-2k = -2 \Rightarrow k = 1$

$4k = 4 \Rightarrow k = 1$

$-2k-2 \Rightarrow -4 \Rightarrow k = 1$

Q18.) If $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Sol. $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$

$I + A =$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$$

(L.H.S.)

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$$

$$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$$

$$(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha + \sin\alpha\tan\alpha/2 & -\sin\alpha + \cos\alpha\tan\alpha/2 \\ -\cos\alpha\tan\alpha/2 + \sin\alpha & \sin\alpha\tan\alpha/2 + \cos\alpha \end{bmatrix}$$

Using, $\tan\alpha/2 = \dfrac{\sin\alpha/2}{\cos\alpha/2}$

$$\sin\frac{\alpha}{2} = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\cos\alpha = 1 - 2\sin^2\alpha/2 = 2\cos^2\alpha/2 - 1$$

$$\Rightarrow \begin{bmatrix} 1 - 2\sin^2\alpha/2 + 2\sin\alpha/2\cancel{\cos\alpha/2}\,\dfrac{\sin\alpha/2}{\cancel{\cos\alpha/2}} & -2\sin\alpha/2\cos\alpha/2 + (2\cos^2\alpha/2 - 1)\dfrac{\sin\alpha/2}{\cos\alpha/2} \\ -(2\cos^2\alpha/2 - 1)\dfrac{\sin\alpha/2}{\cos\alpha/2} + 2\dfrac{\sin\alpha/2}{\cos\alpha/2} & 2\sin\alpha/2\cos\alpha/2 \cdot \dfrac{\sin\alpha/2}{\cos\alpha/2} + (1 - 2\sin^2\alpha/2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 - \cancel{2\sin^2\alpha/2} + \cancel{2\sin^2\alpha/2} & \cancel{-2\sin\alpha/2\cos\alpha/2} + \cancel{2\cos\alpha/2\sin\alpha/2} - \dfrac{\sin\alpha/2}{\cos\alpha/2} \\ \cancel{-2\cos^2\alpha/2 \cdot \dfrac{\sin\alpha/2}{\cos\alpha/2}} + \dfrac{\sin\alpha/2}{\cos\alpha/2} + \cancel{2\sin\alpha/2\cos\alpha/2} & \cancel{2\sin^2\alpha/2} + 1 - \cancel{2\sin^2\alpha/2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix} = R.H.S.$$

L.H.S = R.H.S.

Hence, verified

Q11.) A trust fund has ₹30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹30,000 among the two types of bonds. If the trust fund ~~must~~ must obtain an annual total interest of:

(a) ₹1800 (b) ₹2000

Sol. Total = ₹30,000

Ist part = x

IInd part = $(30,000 - x)$

(a) $\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix} = 1800$

$\Rightarrow \left[\dfrac{5x}{100} + (30000-x)\dfrac{7}{100}\right] = 1800$

$\Rightarrow \left[\dfrac{5x}{100} + \dfrac{210000}{100} - \dfrac{7x}{100}\right] = 1800$

$\Rightarrow \left[\dfrac{-2x + 210000}{100}\right] = 1800$

$-2x + 210000 = 180000$

$\Rightarrow -2x = -30000$

$\Rightarrow x = ₹15,000$; $(30000 - x) = ₹15,000$ Ans

(b) $\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix} = 2000$

$\Rightarrow \left[\dfrac{5x}{100} + \dfrac{210000}{100} - \dfrac{7x}{100}\right] = 2000$

$\Rightarrow \left[\dfrac{-2x + 210000}{100}\right] = 2000$

$\Rightarrow \dfrac{-2x + 210000}{100} = 2000$

$\Rightarrow -2x + 210000 = 200000$

$\Rightarrow -2x = -10000$

$\Rightarrow x = 5{,}000 \;;\; (30000 - x) = 25{,}000$ Ans

EXTRA QUESTION

Q $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 - 5A + 7I = 0$

Using this, find A^4.

Sol. $A^2 = 5A - 7I$

$A^3 = A \cdot A^2$

$= A \cdot (5A - 7I)$

$= 5A^2 - 7IA$

$= 5(5A - 7I) - 7A$

$= 25A - 35I - 7A$

$= 18A - 35I$

$A^4 = A \cdot A^3$

$= A(18A - 35I)$

$= 18A^2 - 35IA)$

$= 18(5A - 7I) - 35A$

$= 90A - 126I - 35A$

$= 55A - 126I$

$= 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} - \begin{bmatrix} 126 & 0 \\ 0 & 126 \end{bmatrix}$

$= \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$ Ans

$\begin{array}{r} 165 \\ -126 \\ \hline 39 \end{array}$

Q20.) The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Sol. 10 dozen chemistry books = $10 \times 12 = 120$ (₹80 each)
8 dozen physics books = $8 \times 12 = 96$ (₹60 each)
10 dozen economics books = $10 \times 12 = 120$ (₹40 each)

$$\text{Total amount} = \begin{bmatrix} 120 & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$$= \begin{bmatrix} 9600 + 5760 + 4800 \end{bmatrix}$$
$$= \begin{bmatrix} 20160 \end{bmatrix}$$
$$= ₹20,160 \text{ Ans}$$

$$\begin{array}{r} 96 \\ \times 6 \\ \hline 576 \end{array} \qquad \begin{array}{r} 9600 \\ 5760 \\ 4800 \\ \hline 20160 \end{array}$$

[Given, orders $X = 2 \times n$, $Y = 3 \times k$, $Z = 2 \times p$, $W = n \times 3$, $P = p \times k$]

Q21.) The restriction on n, k and p so that $PY + WY$ will be defined are:
(A) $k = 3$, $p = n$
(B) k is arbitrary, $p = 2$
(C) p is arbitrary, $k = 3$
(D) $k = 2$, $p = 3$

Sol. $PY + WY$

$$\underbrace{[\quad]_{p\times k}[\quad]_{3\times k}}_{k=3} + [\quad]_{n\times 3} \cdot [\quad]_{3\times k}$$

$$\Rightarrow \underbrace{[\quad]_{p\times k} + [\quad]_{n\times k}}_{p=n}$$

$\therefore$ (A) $k = 3$, $p = n$ Ans

Q22.) If $n=p$, then the order of the matrix $7X-5Z$ is:

(A) $p\times 2$ (B) $2\times n$ (C) $n\times 3$ (D) $p\times n$

Sol. $7\begin{bmatrix} & \end{bmatrix}_{2\times n} - 5\begin{bmatrix} & \end{bmatrix}_{2\times p}$

(B) $2\times n$ Ans

18|4|23

☆ TRANSPOSE OF A MATRIX

If A is any matrix then the matrix obtained by interchanging the columns x rows and rows x columns is called transpose of a matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$A' \text{ or } A^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

- PROPERTIES

(i) $(A+B)' = A' + B'$
(ii) $(A-B)' = A' - B'$
(iii) $(A')' = A$
(iv) $(AB)' = B'A'$
$(ABC)' = C'B'A'$
(v) $(kA)' = kA'$
↳ any non-zero scalar quantity

☆ SYMMETRIC MATRIX

A square matrix A is called symmetric matrix if $A' = A$

$$a_{ij} = a_{ji} \quad , \forall\, i, j$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$\boxed{A' = A}$$

★ SKEW SYMMETRIC MATRIX

A square matrix A is said to be skew symmetric matrix if $A' = -A$

$$a_{ij} = -a_{ji} \quad \forall\ i, j$$

$$A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -4 \\ -3 & 4 & 0 \end{bmatrix}$$

$$A' = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix} = -A$$

THEOREM Show that all the elements on a leading diagonal of a skew symmetric matrix are all zero.

Let A be any square matrix such that $A' = -A$

$$a_{ij} = -a_{ji} \quad \forall\ i, j$$

Put $i = j$

$$a_{ii} = -a_{ii}$$

$$\Rightarrow 2a_{ii} = 0$$

$$\Rightarrow a_{ii} = 0$$

Diagonal elements of a skew symmetric matrix are zero.

THEOREM A matrix which is both symmetric and skew symmetric is a null matrix

Let A be a symmetric matrix

$$A' = A$$

$$a_{ij} = a_{ji} \quad \text{—— (1)} \quad \forall\ i, j$$

Let A also be skew symmetric matrix

$$A' = -A$$

$$a_{ij} = -a_{ji} \quad \text{—— (2)} \quad \forall\ i, j$$

From ① and ②,

$$a_{ij} = -a_{ij} \quad \forall\, i,j$$
$$\Rightarrow 2a_{ij} = 0 \quad \forall\, i,j$$
$$\Rightarrow a_{ij} = 0 \quad \forall\, i,j$$

A is a null matrix

THEOREM For any square matrix A with real number entries, $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix

Given: A is a square matrix

To show: $A + A'$ is symmetric i.e $(A+A')' = A + A'$

$A - A'$ is skew symmetric i.e. $(A-A)' = -(A - A')$

Proof: $(A + A')' = A' + (A')'$

$= A' + A$ (matrix addition is commutative)

$= A + A'$

$A + A'$ is symmetric matrix

$(A - A')' = A' - (A')'$

$= A' - A$

$= A' + (-A)$ (matrix addition is commutative)

$= -A + A'$

$= -(A - A')$

$A - A'$ is skew symmetric matrix.

THEOREM Every square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix

Let A be any square matrix

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

Let $P = \frac{1}{2}(A + A')$, $Q = \frac{1}{2}(A - A')$

To show: $P' = P$ and $Q' = -Q$

Proof: $P = \frac{1}{2}(A + A')$

$P' = \left[\frac{1}{2}(A + A')\right]'$

$= \frac{1}{2}(A + A')'$ $\because (kA)' = kA'$

$= \frac{1}{2}(A' + (A')')$

$= \frac{1}{2}(A' + A)$

$= \frac{1}{2}(A + A')$ $\because$ matrix addition is commutative

P is symmetric matrix

$Q = \frac{1}{2}(A - A')$

$Q' = \left[\frac{1}{2}(A - A')\right]'$

$= \frac{1}{2}(A - A')'$ $\because (kA)' = kA'$

$= \frac{1}{2}(A' - (A')')$

$= \frac{1}{2}(A' - A)$

$= \frac{1}{2}(A' + (-A))$

$= \frac{1}{2}(-A + A')$ $\because$ matrix addition is commutative

$= -\frac{1}{2}(A - A')$

$Q' = -Q$

Q is skew symmetric matrix

$\therefore A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

<u>Uniqueness</u> : Let $A = R + S$

where R is symmetric matrix and S is skew symmetric matrix

$R' = R$ and $S' = -S$

$A' = (R+S)'$

$\Rightarrow A' = R' + S'$

$\Rightarrow A' = R - S$ —— ①

$A = R + S$ —— ②

① + ② $\Rightarrow R = \frac{1}{2}(A + A')$

① − ② $\Rightarrow S = \frac{1}{2}(A - A')$

Hence, A is uniquely expressed as sum of symmetric matrix and skew symmetric matrix.

EXTRA QUESTION [EXAMPLE 24]

Q A and B are symmetric matrices then show that AB is symmetric iff $AB = BA$ i.e A and B commute.

Sol. A and B are symmetric matrices

$A' = A$, $B' = B$

To show : $(AB)' = AB$

Proof : (i) $(AB)' = B'A'$

$= BA$

$\because$ A B commute

$\Rightarrow AB = BA$

$(AB)' = AB$

$\therefore AB$ is symmetric

(ii) $(AB)' = AB$

$(AB)' = B'A'$

$= BA$

But $(AB)' = AB$

$AB = BA$

$\therefore$ A and B commute

19/4/23

~~EXTRA QUESTION [MISC Ques 15 Old Book]~~

~~If A is square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$ is equal to~~

~~(A) A~~ ~~(B) I - A~~ ~~(C) I~~ ~~(D) 3A~~

~~Sol.~~

* ~~EXTRA QUESTIONS~~

EXTRA QUESTION

Q. Express the matrix A as a sum of symmetric and skew symmetric matrix.

$$A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

Sol. $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$$A' = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A')$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 2 & 10 & 7 \\ 5 & 7 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 3 & 5 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & \frac{1}{2} \\ 1 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 3 & \frac{5}{2} \\ 3 & 5 & \frac{7}{2} \\ \frac{5}{2} & \frac{7}{2} & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & \frac{1}{2} \\ 1 & 0 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

19/4/23

☆ EXERCISE - 3.3

Q1.) Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(i) $\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Q2.) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$

Sol. $A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

$(A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$
L.H.S(i)

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, \quad B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\underset{R.H.S(i)}{A' + B'} = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$\underset{R.H.S(ii)}{A' - B'} = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

~~A-B~~

$$A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\underset{L.H.S(ii)}{(A - B)'} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Hence, verified

Q3.) If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$

Sol. $(A')' = A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$(A+B)' =$ L.H.S(i)

$$\begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$(A-B)' =$ L.H.S(ii)

$$\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$A' + B' =$ R.H.S(i)

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$A' - B' =$ R.H.S (ii)

$$\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Hence, verified.

Q4.) If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.

Sol. $A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$$2B = 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$(A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

Q5.) For the matrices A and B, verify that $(AB)' = B'A'$, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(i) $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$(AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$
L.H.S

$A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$, $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$
R.H.S

L.H.S = R.H.S

Hence, verified.

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

(ii) $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$; $(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$
L.H.S.

$A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$, $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$B'A' =$ R.H.S. $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

L.H.S. = R.H.S

hence, verified.

Q6) If

(i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verity that $A'A = I$

(i) $A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow A'A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$\Rightarrow A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

hence, verified

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$.

(ii) $A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$\Rightarrow A'A = \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & \sin\alpha\cos\alpha - \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha - \sin\alpha\cos\alpha & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$

$$\Rightarrow A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, verified.

Q7.) (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(i) $A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

$A = A'$

$\therefore$ A is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

(ii) $A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$-A = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$A' = -A$

$\therefore$ A is a skew symmetric matrix.

Q8.) For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Sol. $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

(i) $(A + A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$(A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$(A + A') = (A + A')'$

Hence, verified.

(ii) $(A - A') = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$-(A - A') = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$(A - A')' = -(A - A')$

Hence, verified.

Q9.) Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$.

Sol. $A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$\frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

$$\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

Let the given matrix be denoted as A

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A')$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A')$$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(ii) Let the given matrix be denoted as A

$$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$P = \frac{1}{2}(A+A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$Q = \frac{1}{2}(A-A') = \frac{1}{2}\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

(iii) Let the given matrix be denoted as A

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A') = \frac{1}{2}\left(\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 5/2 & 3/2 \\ -5/2 & 0 & 3 \\ -3/2 & -3 & 0 \end{bmatrix}$$

(iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

(iv) Let the given matrix be denoted as A

$$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$P = \frac{1}{2}(A+A') = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$$

$$Q = \frac{1}{2}(A-A') = \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Q11.) ~~Choose~~ If A, B are symmetric matrices of same order, then $AB-BA$ is a

(A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

Sol. A and B are symmetric matrices

$\therefore A' = A$

$B' = B$

$$(AB-BA)' = (AB)' - (BA)'$$
$$= B'A' - A'B'$$
$$= BA - AB$$
$$= BA + (-AB)$$
$$= -AB + BA$$
$$= -(AB-BA)$$

$\therefore$ (A) Skew-symmetric matrix Ans

Q12.) If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) π (D) $\frac{3\pi}{2}$

Sol. $A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A + A' = I$

$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{2}$

$\Rightarrow \alpha = 60^\circ = \frac{\pi}{3}$

$\therefore$ (B) $\frac{\pi}{3}$ Ans

19/4/25

☆ EXERCISE - 3.4

Q1) Matrices A and B will be inverse of each other only if

(A) $AB = BA$

(B) $AB = BA = O$

(C) $AB = O$

(D) $AB = BA = I$

19/4/25

★ MISCELLANEOUS EXERCISE ON CHAPTER 3

Q1.) If A and B are symmetric matrices, prove that AB − BA is a skew symmetric matrix.

Sol. $A' = A$

$B' = B$

$(AB - BA)' = (AB)' - (BA)'$

$\Rightarrow (AB - BA)' = B'A' - A'B'$

$\Rightarrow (AB - BA)' = BA - AB$

$\Rightarrow (AB - BA)' = BA + (-AB)$

$\Rightarrow (AB - BA)' = -AB + BA$

$\Rightarrow (AB - BA)' = -(AB - BA)$

Hence, proved.

Q2.) Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Sol. (i) $A' = A$

$(B'AB)' = B'A'B$

$= B'AB$

$\therefore B'AB$ is symmetric

(ii) $A' = -A$

$(B'AB)' = B'A'B$

$= B'(-A)B$

$= -B'AB$

$\therefore B'AB$ is skew symmetric

Q3.) Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

Sol. $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

$A'A = I$

$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz+(-yz)-yz \\ 0-xz+xz & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left.\begin{aligned} 2x^2 = 1 &\Rightarrow x = \pm \frac{1}{\sqrt{2}} \\ 6y^2 = 1 &\Rightarrow y = \pm \frac{1}{\sqrt{6}} \\ 3z^2 = 1 &\Rightarrow z = \pm \frac{1}{\sqrt{3}} \end{aligned}\right\} \text{Ans}$$

Q4.) For what values of x: $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$?

Sol. $\begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 + 4 + 4x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4(x+1) \end{bmatrix} = 0$$

$$4(x+1) = 0$$

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$ Ans

Q5.) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

Sol. $$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow 0$$

Q6.) Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

Sol. $$\begin{bmatrix} x + 0 - 2 & 0 - 10 - 0 & 2x - 5 - 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x - 2 & -10 & 2x - 8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$\Rightarrow \quad [x^2 - 2x - 40 + 2x - 8] = 0$

$\Rightarrow \quad [x^2 - 48] = 0$

$x^2 = 48$

$\Rightarrow x = \pm\sqrt{2\times2\times2\times2\times3}$

$\Rightarrow x = \pm 4\sqrt{3}$ Ans

Q7.) A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Products		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a) If unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(a) Market I

$$\begin{bmatrix}10,000 & 2,000 & 18,000\end{bmatrix}\begin{bmatrix}2.5\\1.5\\1\end{bmatrix}$$

$\Rightarrow [25000 + 3000 + 18000]$

$\Rightarrow [46000]$

₹46,000 Ans

Market II

$$\begin{bmatrix}6,000 & 20,000 & 8,000\end{bmatrix}\begin{bmatrix}2.5\\1.5\\1\end{bmatrix}$$

$\Rightarrow [15000 + 30000 + 8000]$

$\Rightarrow [53000]$

₹53,000 Ans

(b) If the unit costs of the above three commodities are ₹2.00, ₹1.00, 50 paise respectively. Find the gross ~~product~~ profit.

(b) Market I

$$[10,000 \quad 2,000 \quad 18,000]\begin{bmatrix}2\\1\\0.5\end{bmatrix}$$

$\Rightarrow [20000 + 2000 + 9000]$

$\Rightarrow$ ~~[46000]~~ $[31000]$

Gross profit = ₹46,000 − ₹31,000
= ₹15,000 Ans

Market II

$$[6,000 \quad 20,000 \quad 8,000]\begin{bmatrix}2\\1\\0.5\end{bmatrix}$$

$\Rightarrow [12000 + 20000 + 4000]$

$\Rightarrow [36000]$

Gross profit = ₹53,000 − ₹36,000
= ₹17,000 Ans

Q8) Find the matrix X so that $X\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix} = \begin{bmatrix}-7&-8&-9\\2&4&6\end{bmatrix}$.

Sol. $X\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{2\times3} = \begin{bmatrix}-7&-8&-9\\2&4&6\end{bmatrix}_{2\times3}$

X will be 2x2 matrix

Let $X = \begin{bmatrix}u&w\\v&x\end{bmatrix}$

$$\begin{bmatrix}u&w\\v&x\end{bmatrix}\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix} = \begin{bmatrix}-7&-8&-9\\2&4&6\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}u+4w & 2u+5w & 3u+6w\\ v+4x & 2v+5x & 3v+6x\end{bmatrix} = \begin{bmatrix}-7&-8&-9\\2&4&6\end{bmatrix}$$

$v + 4w = -7$ — ①

$2v + 5w = -8$ — ②

$3v + 6w = -9$ — ③

$v + 4x = 2$ — ④

$2v + 5x = 4$ — ⑤

$3v + 6x = 6$ — ⑥

① × 2 − ②

$$\begin{aligned} \cancel{2v} + 8w &= -14 \\ \cancel{2v} + 5w &= -8 \\ - \quad - \quad &\quad + \\ \hline 3w &= -6 \end{aligned}$$

$\boxed{w = -2}$

① ⇒ $v + 4(-2) = -7$

⇒ $v - 8 = -7$

⇒ $\boxed{v = 1}$

④ × 2 − ⑤

$$\begin{aligned} \cancel{2v} + 8x &= 4 \\ \cancel{2v} + 5x &= 4 \\ - \quad - \quad &\quad - \\ \hline 3x &= 0 \end{aligned}$$

$\boxed{x = 0}$

④ ⇒ $v + 4(0) = 2$

⇒ $\boxed{v = 2}$

$\therefore X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ Ans

Q9.) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$
(C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

Sol. $A^2 = A \cdot A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \beta\alpha \\ \gamma\alpha - \alpha\gamma & \gamma\beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\alpha^2 + \beta\gamma = 1$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

$\therefore$ (C) $1 - \alpha^2 - \beta\gamma$ Ans

Q10.) If the matrix is both symmetric and skew symmetric, then

(A) A is a diagonal matrix (B) A is a zero matrix
(C) A is a square matrix (D) None of these

Q11.) If A is square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$ is equal to

(A) A (B) $I - A$ (C) I (D) 3A

Sol. $(I+A)^2 = (I+A)(I+A)$
$= I^2 + IA + IA + A^2$
$= I + A + A + A$
$= I + 3A$

$(I+A)^3 = (I+A)^2 (I+A)$
$= (I+3A)(I+A)$
$= I^2 + IA + 3AI + 3A^2$
$= I + A + 3A + 3A$
$= I + 7A$

$(I+A)^3 - 7A \Rightarrow I + 7A - 7A \Rightarrow I$ $\therefore$ (C) I Ans